Full research paper

DATA ENVELOPMENT ANALYSIS AS A TOOL FOR IDENTIFYING REFERENCE SCHOOLS: A STUDY IN THE CONTEXT OF HIGH SCHOOL IN A BRAZILIAN STATE

Denilson Junio Marques Soares[1]✉
Wagner dos Santos[2]

[1]Federal Institute of Education, Science and Technology of Minas Gerais, Brazil

[2]Federal University of Espírito Santo, Brazil

✉ denilson.marques@ifmg.edu.br

ABSTRACT

To ensure compliance with students' educational rights and monitor education systems, many countries have developed indicators that assign statistical value to the quality of education. The Basic Education Development Index (Ideb) is widely used in Brazil as a strategic management tool. This study proposes using Ideb, weighted by contextualized educational measures, to identify schools that excel compared to others with similar characteristics (referred to as reference schools). The study employs the output-oriented Variable Returns to Scale approach of Data Envelopment Analysis, along with the Malmquist Index, to assess changes in outcomes over two editions of the indicator. The findings show that, in 2017, 17 schools operated on the production frontier and had at least one partner of excellence. By 2019, this number increased to 18 out of the 222 schools analyzed. Applying the Malmquist Index further indicated that most schools experienced modest improvements in technical efficiency during the analyzed period, effectively utilizing resources to achieve similar or better outcomes. This study underscores the importance of understanding successful school strategies, providing valuable insights for educational improvements, and facilitating the adoption of effective methods in comparable institutions.

KEYWORDS

Basic Education Development Index, Data Envelopment Analysis, educational efficiency, reference schools, Brazil

HOW TO CITE

Soares D. J. M., dos Santos W. (2024) 'Data Envelopment Analysis as a Tool for Identifying Reference Schools: A Study in the Context of High School in a Brazilian State', *Journal on Efficiency and Responsibility in Education and Science*, vol. 17, no. 4, pp. 314–327. http://dx.doi.org/10.7160/eriesj.2024.170404

Article history
Received
April 26, 2024
Received in revised form
August 27, 2024
Accepted
October 14, 2024
Available on-line
December 31, 2024

Highlights

- *The article presents an alternative to using the Basic Education Development Index in Brazil to identify reference schools, weighting it by educational measures to evaluate the context of basic education.*
- *The study reveals that in 2017, 17 schools operated on the production frontier and had at least one partner of excellence. By 2019, this number had increased to 18 out of the 222 schools analyzed.*
- *The application of the Malmquist Index revealed that most schools experienced a modest improvement in their technical efficiency during the analyzed period, managing to use resources more effectively to achieve similar or better outcomes.*
- *This study emphasizes the need to understand successful school strategies, offering vital insights for educational improvements and facilitating the adoption of effective methods in comparable institutions.*

INTRODUCTION

The educational landscape has significantly transformed the role and importance attributed to external assessments in recent years. These have become instruments for measuring school performance and a fundamental component in formulating and implementing educational policies at national and international levels. In this context, external evaluation has transcended its traditional role, acquiring strategic relevance in implementing initiatives to guarantee the right to education (Rutkowski et al., 2020; Jimenez and Modaffari, 2021; Ehren, 2021).

In Brazil, the pioneering initiative for a comprehensive analysis of the educational system emerged in the 1990s with the creation of the Basic Education Assessment System (Saeb). It is worth noting that the Brazilian Federal Constitution, promulgated in 1988, recognizes education as a universal right (Brasil, 1988). Saeb's performance and learning outcomes provide the state with evidence of the fulfillment of this right, especially regarding the training of students through the standardized assessments that comprise it, but also considering the academic trajectory of these subjects based on information produced through contextual questionnaires applied since 1995. As expressed by Soares and Xavier (2013), in the absence of a system like this, the subjective public right established in the constitutional text cannot be monitored and, eventually, demanded.

In fact, the emergence of Saeb allowed society to better understand Brazilian educational outcomes in a general context. However, due to its initially sample-type nature, it was impossible to highlight the particularities of each school and municipality, limiting the development of public policies that could meet their specificities. The system underwent a substantial restructuring in 2005 to overcome this situation, resulting in the creation of the National Assessment of School Performance (known as Prova Brasil), which assumed a census application, i.e., with universal coverage for all students in their final years of each cycle of elementary education in public schools (Brasil, 2005).

Thus, Prova Brasil was created to produce metadata on the education offered by schools and municipalities, assisting educational managers in decision-making and allocating technical and financial resources. In light of this assessment, in the national context, there was the establishment of an educational policy based on the accountability of schools and the actors who work in them, who now have to be held accountable (Fernandes and Gremaud, 2020; Jerrim and Sims, 2022) of the outcomes achieved for society, a practice materialized by the document that established the Education Development Plan (PDE), officially launched by the MEC on April 24, 2007. According to Brasil (2007: 11), the PDE established 'unprecedented connections between evaluation, financing, and management, which invoke a concept that was previously absent from our educational system: accountability and, as a result, social mobilization'.

Among the actions proposed by the PDE, we highlight the Target Plan *Compromisso Todos pela Educação*, implemented through Decree No. 6,094, of April 24, 2007 (Brasil, 2007). From it, and inspired by the United States Educational Responsibility Law (No Child Left Behind) (Soares et al., 2023), it was possible to outline measurable parameters for the quality of education in Brazil, given the consolidation of targets for all teaching modalities and in all administrative spheres, supporting the creation of the Basic Education Development Index (Ideb) for its follow-up and monitoring. In the words of Soares (2016: 149), from this movement, 'Brazilian society understood that education has outcomes and that rights without learning outcomes are mere utopia'.

Ideb is a statistical indicator that combines student performance in the Saeb with flow indicators (promotion, retention, and dropout) produced by the School Census. The biennial publication of its outcomes, together with the pre-established targets, gave rise to a practice that, although unofficial, encouraged the ranking of educational units based solely on the performance achieved in the indicator, as advised in several studies in the specialized literature (Richter and Vieira, 2021; Akkari, 2021; Ranieri, 2024).

In summary, in the Brazilian context, schools with the highest scores began to be recognized as a *reference* for the others, disregarding the multiple facets of the Brazilian educational system, assumed to be standardized and uniform. On the other hand, units with lower grades were marginalized and discredited in society, bringing dire and harmful consequences to education. Contrary to this movement, we start from the hypothesis that reconceptualizing this pre-existing idea of reference schools, starting to consider the characteristics of students, the plurality of educational contexts, and the tensions that shape school reality, can corroborate a more systemic analysis of the outcomes of indicators such as Ideb. This practice has been defended by researchers in different countries and at different times, who demonstrated, in their studies, that school performance reflects these factors in a considerably direct way, requiring controlling their heterogeneities before proposing any comparison between the schools in the quest to verify the success of specific educational policies (Henry et al., 2020; Liu et al., 2020; Brown, 2022; Vermunt et al., 2023).

In this regard, this study seeks to answer the following question: Which schools have better educational outcomes in the Ideb than expected, considering their contextual and general working conditions? To answer this question and guide this investigation, this article aims to develop a final reference indicator capable of classifying the relative performance of a set of schools considered as units of analysis. This classification will take into account not only the Ideb of these units but also educational measures that assess the context of basic education. The empirical case of this study includes all high schools administered by the State of Espírito Santo, located in the southeast region of Brazil, using cross-sectional data from the 2017 and 2019 school years. Data Envelopment Analysis (DEA) was used as the methodological approach.

It is important to highlight that, as noted by Koronakos (2019), the use of DEA to evaluate school effectiveness, particularly when schools are treated as Decision-Making Units (DMUs), remains relatively uncommon, especially within the Brazilian context. This perspective is further supported by Soares et al. (2022), who observes that no previous studies in Brazil employ DEA to analyze school effectiveness at the level of basic education schools. This endows our study with a pioneering character due to the innovative application of DEA in a new context and for fostering a deeper understanding of how contextual variables influence school effectiveness. This understanding has the potential to assist in the formulation of more effective educational policies.

In addition to this introduction, the article is structured into four sections. First, a brief theoretical reference is provided on the Ideb, the Educational Indicators for Context Assessment in Basic Education used in the study, and DEA. Next, the materials and methods are detailed. The subsequent sections present the results obtained, followed by a discussion. The article concludes with a summary of the key findings and the implications drawn from the study.

LITERATURE REVIEW

Overview of the Brazilian Basic Education System

In Brazil, the Ministry of Education (MEC) is the federal body responsible for formulating policies and guidelines for basic education. In contrast, the implementation and management of these policies are shared between state and municipal governments. The Brazilian basic education system is structured into three main stages: early childhood education, elementary school, and high school. Early childhood education serves children up to 5 years old and includes day care and preschool. Elementary school is divided into two segments: the early years (1st to 5th grade) for children aged 6 to 10 and the final years (6th to 9th grade) for students aged 11 to 14. High school serves students aged 15 to 17 and spans three years. Public schools account for the majority of enrollments in basic education, although private institutions also play a significant role, particularly in urban areas (Brasil, 2021).

Despite educational advancements, such as increased enrollment rates and the implementation of conditional cash transfer programs like Bolsa Família (Gangopadhyay, 2020), the Brazilian education system continues to face substantial challenges in ensuring quality education for all students. Regional and socioeconomic disparities still profoundly impact access to education and educational outcomes, highlighting the need for more effective public policies adapted to the diverse realities across the country (Ernica et al., 2024).

The recent release of the 2023 IDEB results has brought critical reflections on the educational performance of Brazil. The data revealed that the national goal set for the first cycle of the indicator (2007-2021) was only achieved in the early years of elementary school. However, in the final years of elementary and high school, the results fell short of the projected targets (Brasil, 2024). This discrepancy signals a challenging scenario, indicating that there is still a long way to go to improve the quality of education in the more advanced stages of basic education.

This challenge is also evidenced by Brazil's performance in the Programme for International Student Assessment (PISA), conducted by the Organisation for Economic Co-operation and Development (OECD). PISA, conducted every three years, assesses 15-year-old students in the areas of Reading, Mathematics, and Science. The results of PISA 2022, presented in Figure 1, reflect the performance of South American countries, comparing them with the OECD average, and underscore the urgent need for educational reforms to raise the quality of education in Brazil (OECD, 2023).

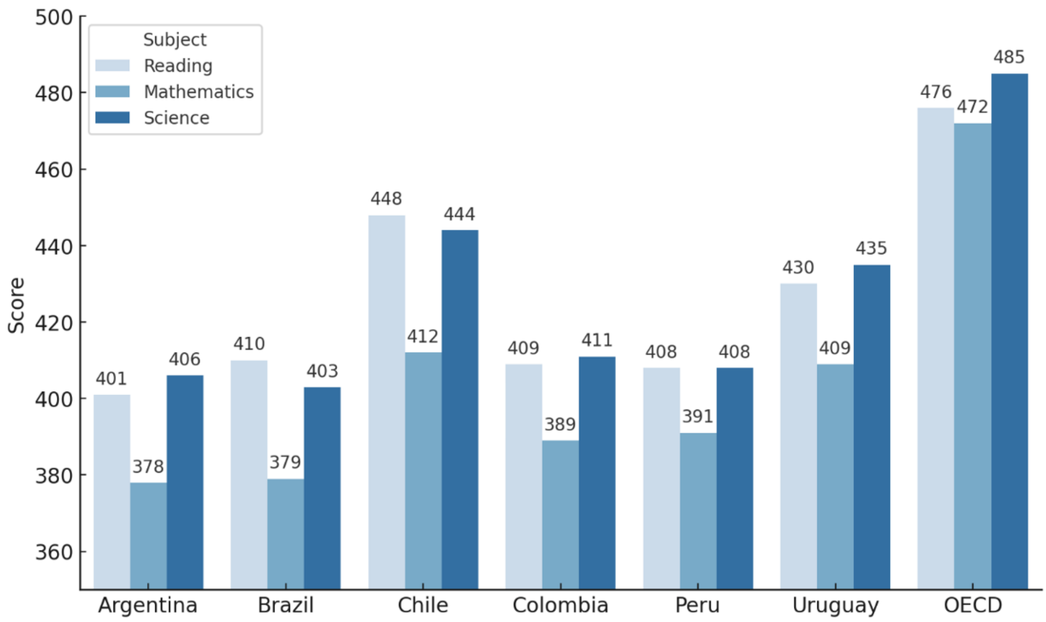

Figure 1: PISA 2022 Results for South American Countries and the OECD Average by Subject (own elaboration based on data provided by the OECD, 2023)

Figure 1 reveals Brazil's poor performance in PISA 2022, where the country ranks last in Science and Mathematics among the South American countries evaluated. This result reflects not only the educational system itself but also the deep social inequality present in the country, which translates into educational inequality. Indeed, numerous studies indicate that the context in which schools are embedded—including factors such as socioeconomic level, access to resources, and school infrastructure—directly influences their educational outcomes (Carvalhaes et al., 2023; Queiroga et al., 2024; Oliveira et al., 2024; Soares and Santos, 2024). Schools tend to face greater challenges in more vulnerable regions, where living conditions are more precarious, resulting in lower performance (Queiroga et al., 2024).

Acknowledging this reality, there is a growing movement among Brazilian researchers aimed at analyzing the effectiveness of schools operating in these adverse contexts (Paschoalotto et al., 2020; Moraes et al., 2021; Silva, 2021; Dias and Zouain, 2023). The goal is to identify schools that, despite the difficulties, manage to achieve superior educational outcomes. These analyses are crucial for understanding how these schools overcome the barriers imposed by their contexts and how their practices can be replicated in other institutions to reduce educational inequality and improve national performance.

Applications of DEA in Efficiency Evaluation

DEA is a non-parametric methodology used to evaluate the relative efficiency of DMUs that convert multiple inputs into multiple outputs (Ferreira and Gomes, 2020).

Initially developed by Charnes et al. (1978), DEA is based on mathematical models of linear programming to create an efficiency frontier against which the performance of DMUs is assessed.

The main value of DEA lies in its ability to evaluate the efficiency of DMUs holistically, considering multiple inputs and outputs without the need to specify a predetermined functional form between them (Flegl et al., 2023). Unlike parametric methods, such as linear regression, which require an explicit assumption about the relationship between variables, DEA allows the data to "speak for itself", identifying the most efficient units operating on the efficiency frontier (Wyszynski, 2024).

In the scientific literature, DEA has been widely used to evaluate the efficiency of organizations and institutions, including hospitals (Pereira et al., 2021; Hajiagha et al., 2023; Zubir et al., 2024), airports (Özsoy and Örkcü, 2021; Zeng et al., 2024), banks (Li et al., 2022; Ullah et al., 2023), local governments (Ryan et al., 2021; Yin et al., 2021), and sports organizations (Guzmán-Raja and Guzmán-Raja, 2021; Gökgöz and Yalçı, 2022; Wyszynski, 2024), among others. This broad applicability highlights the flexibility of DEA in assessing performance across a wide range of sectors, each with its unique set of inputs and outputs.

Application of DEA in Education

The DEA approach has been widely adopted in the educational field, particularly due to its simplicity and effectiveness in contexts involving multiple inputs and outputs, which are typical of the educational environment (Johnes, 2006). The theoretical foundation of these studies holds that school inputs are crucial determinants for achieving educational outcomes (Lima, 2011). Furthermore, Liu et al. (2013) identified that the application of DEA in education ranks among the top five utilization domains, underscoring its relevance and popularity in this field.

For instance, recent publications in the educational field have extensively explored the efficiency of higher education institutions using DEA as a methodological tool. Studies have examined institutions across various countries, including Spain (Salas-Velasco, 2020), Mexico (Moncayo–Martínez et al., 2020), Brazil (Santos Tavares et al., 2021), Canada (Ghimire et al., 2021), China (Jiang et al., 2020; Chen et al., 2021), Colombia (Zuluaga et al., 2023), Turkey (Mammadov and Aypay, 2020; Doğan, 2023), and the Czech Republic (Hančlová and Chytilová, 2023).

Additionally, Popović et al. (2020) used a combination of DEA and Multi-Criteria Decision-Making Methods to evaluate professors' performance at the Faculty of Organizational Sciences, University of Belgrade, Serbia. On the other hand, De la Hoz et al. (2021) applied DEA to assess and predict academic efficiency across 256 engineering programs at universities in Colombia.

From an international comparative perspective, Torres-Samuel et al. (2020) employed DEA to analyze fifteen Latin American countries, considering factors that reflect their progress in research and development, science and technology, education, and innovation. In the context of the European Union, the studies by Dincă et al. (2021) and Halásková et al. (2020) offer valuable insights. The first study seeks to analyze the efficiency of the educational sector at different levels of education. In contrast, the second focuses on evaluating the efficiency of research and development in the public and private sectors in the region. Additionally, from a comparative perspective, Sinuany-Stern and Hirsh (2021) assess the relative efficiency of higher education in OECD countries, considering how these countries utilize their public resources to achieve results compared to their peers.

Regarding basic education, DEA analysis has also been used to evaluate the efficiency of different educational systems around the world. Ramzi et al. (2016) analyzed the efficiency of primary and secondary education in 24 provinces of Tunisia. Halásková et al. (2022) investigated the efficiency of 26 public and private secondary schools in Slovakia. In the Latin American context, Delprato and Antequera (2021) applied a DEA model to compare the efficiency between public and private secondary schools, highlighting the gaps existing in the region. In Mexico, Flegl et al. (2023) explored the efficiency of the educational system and its impact on regional, economic, and social development. In Portugal, Silva et al. (2020) evaluated the performance of secondary schools based on students' outcomes at the end of their first year of university, using data from the University of Porto and the Catholic University of Porto.

These studies collectively emphasize the value of DEA in identifying best practices and areas for improvement within the education system. By providing a comparative analysis of school efficiency, DEA enables policymakers and educators to better understand the factors contributing to successful educational outcomes, even in contexts marked by adversity.

MATERIALS AND METHODS

Data Envelopment Analysis

Data Envelopment Analysis (DEA) is a multivariate technique based on non-parametric mathematical models designed to evaluate the performance of a group of entities known as Decision-Making Units (DMUs). Utilizing linear programming tools, DEA calculates the performance of each DMU in comparison to all other DMUs in the set, establishing an efficient frontier that delineates the top-performing units (Flegl et al., 2023). This analysis identifies necessary adjustments for DMUs operating below this frontier to achieve efficiency through radial projection.

Mathematically, the problem of maximizing the efficiency of a DMU can be expressed as follows:

$$\text{Maximize}\,\theta_0 = \frac{\sum_{r=1}^{s} u_r y_{r0}}{\sum_{i=1}^{m} v_i x_{i0}}$$

Subject to:

$$\frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}} \leq 1, \forall j = 1, 2, \cdots, n$$

$$u_r \geq 0, \forall r$$

$$v_i \geq 0, \forall i$$

Electronic ISSN 1803-1617 Printed ISSN 2336-2375

Where:

- θ_0 is the relative efficiency of the DMU under evaluation.
- y_{rj} is the quantity of the r-th output produced by the j-th DMU.
- x_{ij} is the quantity of the i-th input used by the j-th DMU.
- u^r and v_i are the weights assigned to the outputs and inputs, respectively.

The goal is to maximize the efficiency θ_0 for each DMU under the constraint that all other DMUs' efficiency is less than or equal to 1. When the efficiency $\theta_0 = 1$, the DMU is considered efficient relative to the others in the set. Otherwise, it is considered inefficient.

Two classical DEA approaches stand out: Constant Returns to Scale (CRS) and Variable Returns to Scale (VRS). These approaches differ concerning the proportionality axiom. In the CRS model, any variation in inputs results in a proportional variation in outputs, whereas in the VRS model, this proportionality may not hold true (Ferreira and Gomes, 2020). Both approaches allow for two radial orientations to determine the efficient frontier: input-oriented and output-oriented. The input-oriented approach aims to identify the extent to which inputs can be reduced while maintaining the same level of outputs. In contrast, the output-oriented approach seeks to maximize outputs without increasing the amount of inputs used (Avilés-Sacoto et al., 2021).

Given that relationships in the educational field often do not exhibit constant returns to scale and that, due to budgetary constraints, there is a frequent emphasis on maximizing outputs with existing inputs, this study adopts the DEA-VRS approach, which is focused on outputs.

Malmquist Index

The Malmquist Index is a productivity measure used to assess the performance changes of Decision-Making Units (DMUs) over time. Named after the Swedish economist Sten Malmquist, this index is particularly useful for analyzing DMUs' efficiency changes between two periods by decomposing the total factor productivity change into two components: efficiency change and technological change (Xu et al., 2021).

Mathematically, the Malmquist Index is calculated as follows:

$$M(t,t+1)=\sqrt{\left(\frac{D_t\left(x_{t+1},y_{t+1}\right)}{D_t\left(x_t,y_t\right)}\right)\times\left(\frac{D_{t+1}\left(x_{t+1},y_{t+1}\right)}{D_{t+1}\left(x_t,y_t\right)}\right)}$$

Where:

- $M(t, t + 1)$ is the Malmquist Index between periods t and $t + 1$.
- $D_t(x_t, y_t)$ represents the distance function at time t, which measures the efficiency of a DMU using the technology available at time t.
- $D_t(x_t + 1, y_t + 1)$ represents the distance function at time t, measuring the efficiency of a DMU in the future period but using the technology available at time t.
- $D_{t+1}(x_t, y_t)$and $D_{t+1}(x_t + 1, y_t + 1)$ are the corresponding distance functions at time $t + 1$.

The Malmquist Index can be decomposed into two primary components: Efficiency Change (EC), which captures the shift in the relative efficiency of a DMU between two periods, reflecting whether the DMU has moved closer to or farther from the efficiency frontier, and Technological Change (TC), which measures the shift in the technology frontier itself, indicating whether the overall production possibility set has improved or regressed (Er-Rays and M'dioud, 2024).

If the Malmquist Index is greater than 1, it indicates an improvement in productivity; a value below 1 suggests a decline in productivity. This index is particularly valuable in longitudinal studies, where the goal is to assess the progress or regression of DMUs over time. In this article, the Malmquist Index was used to analyze the changes in the efficiency of school units between the years 2017 and 2019.

Data

This study encompasses an empirical analysis of 222 schools administered by the State of Espírito Santo, Brazil, which offered at least one high school class during the years 2017 and 2019 and had scores in the Basic Education Development Index (Ideb) for this stage. According to the School Management System, these 222 schools represent approximately 79% of the total institutions administered by the State.

The data utilized were obtained from the electronic portal of the National Institute for Educational Studies and Research Anísio Teixeira (Inep), a federal agency linked to the Ministry of Education of Brazil (Inep/MEC) (Brasil, 2024). For the analysis, four educational indicators developed by Inep/MEC were considered as inputs: age-grade distortion rate (TDI), percentage of students declared as white (PCB), socioeconomic level of the school (INSE), and teaching effort (IED).

The TDI refers to the proportion of students who are two or more years behind in their studies. The INSE reflects the socioeconomic level of the students, considering information on family income, parental education, and possession of durable goods collected through contextual questionnaires from the Basic Education Assessment System (Saeb). The PCB measures the proportion of students who self-identify as white. Meanwhile, the IED assesses teaching effort, taking into account variables such as workload, the number of schools where teachers work, and the stages of education in which they are involved. Table 1 presents the descriptive statistics of these contextual indicators.

Statistic	INSE	TDI	PCB	IED
Minimum	4.070	4.100	4.023	0.000
Average	4.810	25.900	33.548	25.130
Standard deviation	0.279	10.418	17.531	16.148
Coef. of variation	5.810%	40.220%	52.260%	64.260%
Maximum	5.460	60.400	88.000	70.000

Table 1: Descriptive statistics of related context indicators (own elaboration)

The Ideb scores for high schools from 2017 and 2019 were used as outputs in two applications of the DEA model, both employing the same inputs. The goal was to assess the variations in the overall efficiency growth of the analyzed school units using the Malmquist Index.

In both these editions of Ideb, schools in the State of Espírito Santo achieved the highest scores for high school among Brazilian states. In 2017, the state's score was 4.400, compared to the national average of 3.800. By 2019, the state's Ideb rose to 4.800, while the national average increased to 4.200. However, despite these improvements, the results fell short of meeting the targets set for the respective years, which were 5.100 and 5.300.

It is important to highlight that the input indicators used in this analysis were selected based on the study by Soares and Santos (2024), which demonstrated through multiple linear regression analysis that these variables collectively explain 50.8% of the variations in the 2019 Ideb.

Additionally, all indicators showed a statistically significant correlation with Ideb, as illustrated in Figure 2, which presents the frequency distribution of these contextual indicators, scatter plots and correlation coefficients, showing pairwise relationships among the indicators and between each indicator and the Ideb of the evaluated school units.

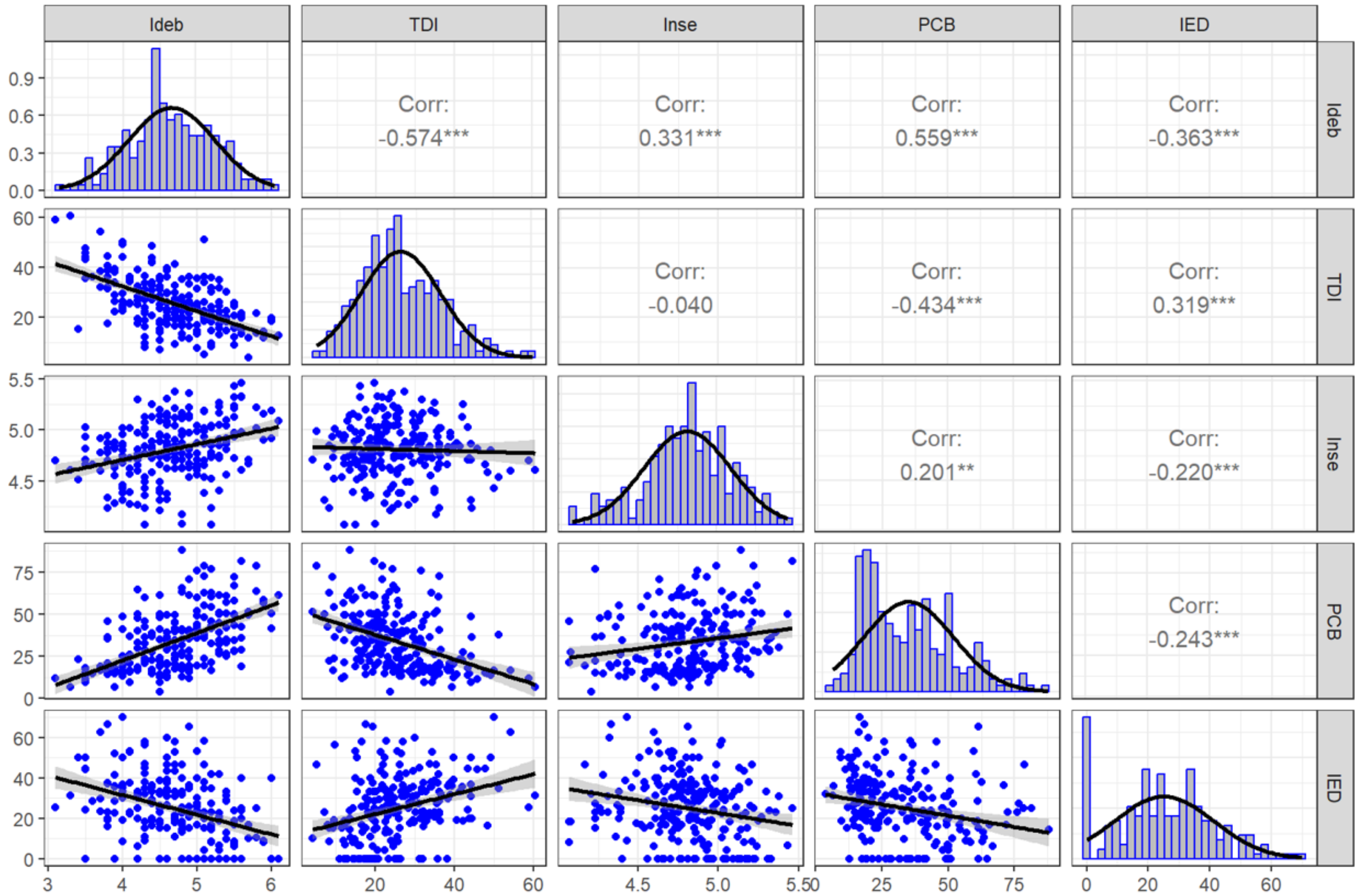

Figure 2: Graphs of frequency distribution, dispersion, and correction coefficients between context indicators related to schools and Ideb (own elaboration based on data provided by the Brasil, 2024)

Notably, the negative sign attributed to TDI and IED indicates an inverse linear relationship with Ideb. Thus, the higher a school's TDI or IED, the lower its Ideb tends to be, on average, as expected. On the other hand, the relationship between the INSE and PCB indicators is direct.

Computational Workflow and Data Transparency

The analyses were conducted using R software, version 4.4.2 (R Core Team, 2023), widely recognized for its statistical analysis and modeling robustness. A significance level of 5% was adopted, ensuring that the results obtained were statistically significant. All source codes used in this study and the database are available in a GitHub repository (https://github.com/denilsonjms/PhD-Thesis/tree/master/Chapter%204). This availability aims to facilitate the replication of the results and allow further analyses by other researchers interested in the topic.

RESULTS

Overall Performance of the Analyzed Schools

The analysis using the DEA method revealed that the schools investigated could potentially increase their 2017 Ideb scores by up to 19.400% while maintaining the same contextual characteristics ($\phi \approx 1.194$). The efficiency of the Decision-Making Units (DMUs), under the variable returns to scale approach, ranged from 58.550% to 100%, with an average of 84.990% and a standard deviation of 9.970%. Figure 3 shows the distribution of efficiency scores among the analyzed units in 2017.

Figure 3: Efficiency scores distribution using 2017 Ideb results as output (own elaboration)

For the model that utilized the 2019 Ideb results as the output, the DEA analysis indicated that the schools could achieve an increase of up to 14.200% in their results while maintaining the same contextual characteristics ($\phi \approx 1.142$). The efficiency of the DMUs in this scenario ranged from 67.320% to 100%, with an average of 88.333% and a standard deviation of 8.111%. Figure 4 presents the distribution of efficiency scores among the analyzed school units in 2019.

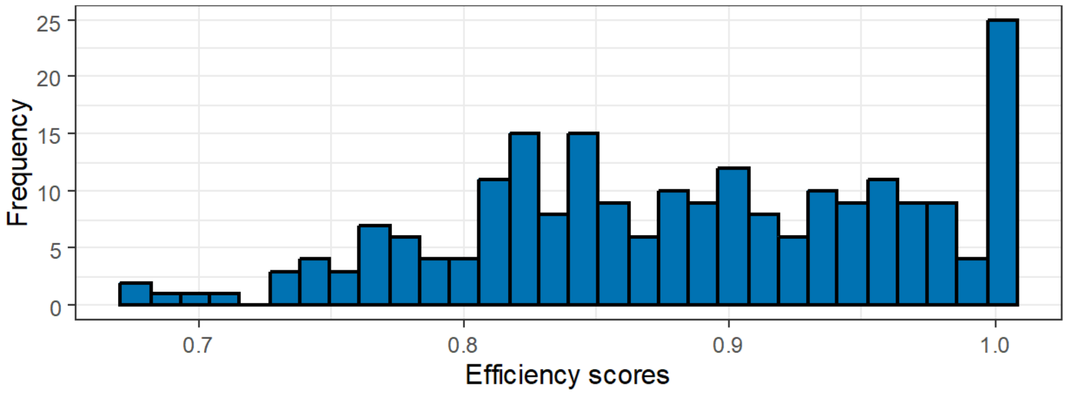

Figure 4: Efficiency scores distribution using 2019 Ideb results as output (own elaboration)

A descriptive analysis of the overall performance efficiency of the analyzed units in each case reveals a slight improvement in average efficiency over time. However, a significant potential for further improvement remains evident among the schools.

Reference Schools

The conducted DEA analysis identified the units operating on the efficiency frontier, meaning those that achieved the maximum technical efficiency score (ϕ =1). In 2017, 21 schools were identified in this condition, representing 10.810% of the analyzed DMUs. Among these, 16 schools (75%) stood out as reference units (reference schools), being considered benchmarks or peers for at least one other school. These schools serve as models of efficiency, offering practices and strategies that can be replicated by other institutions seeking improvement. In 2019, the analysis revealed an increase in the number of efficient units, with 24 schools located on the production frontier, again representing 10.810% of the analyzed DMUs. Among these, 18 schools (75%) were identified as reference units (reference schools), reaffirming their prominent role as benchmarks for other institutions. It is important to highlight that of the 21 schools on the efficiency frontier in 2017, 13 maintained this status in 2019, demonstrating significant consistency in their practices and results over time.

Table 2 presents the schools operating at the efficiency frontier based on the model that uses the 2019 edition of the Ideb. These reference schools are identified along with the number of partner schools. Additionally, the table includes the municipality where these schools are located, the Regional Education Superintendency (SRE) to which they belong, the Ideb scores achieved in the referenced year, and the educational context indicators used as inputs in the analyses.

	Code	School	Municipality	SRE	P.E.	IDEB	TDI	INSE	IED	PCB
1	32037163	EEEFM Francisco Nascimento	Serra	Carapina	126	5.600	15.000	4.980	40.000	18.770
2	32059310	EEEFM Agostinho Agrizzi	Vargem Alta	Cachoeiro de Itapemirim	105	6.000*	18.700	4.920	40.000	50.720
3	32045360	EEEFM Professor Pedro Simao	Alegre	Guaçuí	86	5.100*	51.200	4.540	35.000	37.770
4	32010745	EEEFM Profa. Carolina Pichler	Colatina	Colatina	63	4.800**	35.400	4.170	46.700	19.310
5	32066333	EEEFM Zumbi dos Palmares	Serra	Carapina	51	4.600**	39.700	4.840	50.000	12.540
6	32078315	EEEM Sobreiro	Laranja da Terra	Afonso Cláudio	48	5.200*	14.900	4.070	8.300	45.740
7	32010702	CEEMTI Conde de Linhares	Colatina	Colatina	48	6.000**	19.900	5.190	0.000	41.500
8	32021550	EEEFM Joao Neiva	João Neiva	Linhares	46	5.400**	30.300	4.910	31.300	26.430
9	32079230	EEEM Mario Gurgel	Vila Velha	Vila Velha	42	4.400*	48.400	4.580	16.200	14.680
10	32003005	EEEFM Ecoporanga	Ecoporanga	Barra de São Francisco	24	4.500*	23.700	4.210	32.100	4.020
11	32078528	EEEM Santina Morosini Cupertino	Linhares	Linhares	16	4.000	50.000	4.430	70.000	16.670
12	32015089	EEEFM DE MUCURICI	Mucurici	Nova Venécia	9	4.600*	30.000	4.320	60.000	20.090
13	32040288	EEEM Gomes Cardim	Vitória	Carapina	6	3.300	60.400	4.610	31.300	6.710
14	32049536	CEEFMTI Braulio Franco	Muniz Freire	Guaçuí	6	6.100*	13.000	5.090	0.000	61.520
15	32052618	EEEFM Profa. Hosana Salles	Cachoeiro de Itapemirim	Cachoeiro de Itapemirim	6	5.100*	22.800	5.020	58.300	35.800
16	32079214	EEEM Profa. Maria Candido Kneipp	Muniz Freire	Guaçuí	5	5.200*	24.400	4.230	27.300	76.740
17	32048459	EEEFM Bernardo Horta	Irupi	Guaçuí	4	4.900*	19.400	4.810	65.400	61.550
18	32051840	EEEFM Horacio Plinio	Bom Jesus do Norte	Guaçuí	1	3.800	44.400	4.230	20.000	36.620
					Average	4.920	31.200	4.620	35.110	32.620
					Standard deviation	0.770	14.600	0.360	21.100	20.700
					Coefficient of variation (in %)	15.610	46.800	7.800%	60.100	63.450

* the target was achieved; **there was no projected target

Table 2: High school reference schools in the State of Espírito Santo (own elaboration)

The reference schools identified in Table 2 obtained a 2019 Ideb for secondary education ranging from 3.300 to 6.100, with an average of 4.920, surpassing the State's state schools (4.600) and the proposed target for the year (4.700). The coefficient of variation between grades is classified as medium, according to the classification proposed by Gomes (1985). Of the 14 units that had targets projected for 2019 Ideb, we found that ten achieved or exceeded the objectives, representing a percentage of 71.430%.

Malmquist Index

The Malmquist Index showed a minimum value of 0.864, indicating that some units experienced reduced productivity. In contrast, the maximum value of 1.402 suggests that other units increased their productivity by up to 40.200%. The median of 1.012 reflects a slight productivity gain for most school units. It is observed that 25% of the units maintained their productivity constant (1st quartile equal to 1), while another 25% showed significant improvement, with a productivity increase of more than 5.999% (3rd quartile). Figure 5 illustrates the distribution of the Malmquist Index, highlighting these variations.

The decomposition of the Malmquist index revealed considerable variation in the technical efficiency of the school units over the analyzed period, as indicated by the Efficiency Change Index, which ranged from 0.863 to 1.147. A median of 1.005 suggests that most schools saw a modest improvement in their technical efficiency, managing to utilize resources more effectively to achieve similar or better outcomes.

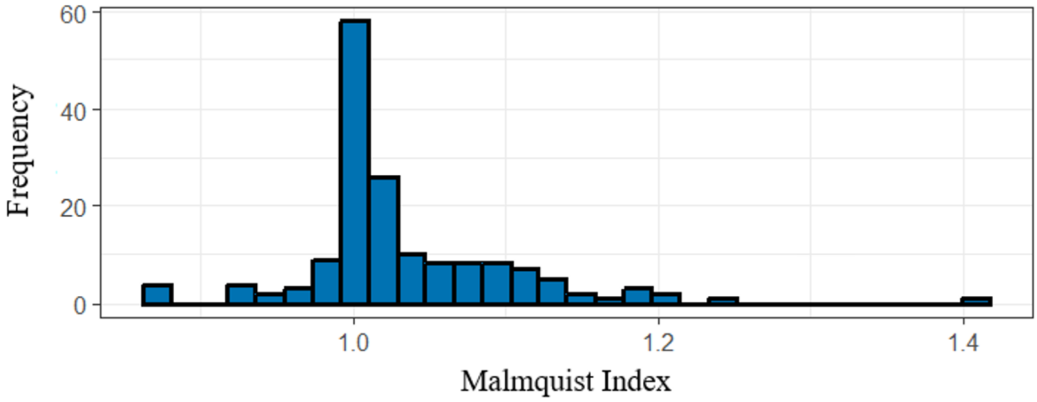

Figure 5: Distribution of the Malmquist Index (own elaboration)

However, the minimum value of 0.863 indicates that some schools experienced a decline in efficiency, likely due to internal challenges or contextual factors they could not address. Meanwhile, with a median of 1, the Technological Change Index points to a general stabilization in the technological production frontier, with half of the units showing no significant technological changes.

Overall, the findings suggest a slight improvement in schools' technical efficiency over time, with some units making noteworthy progress. However, technological progress was uneven, with certain schools advancing more rapidly than others. This indicates that while there is a general trend towards improvement, disparities remain that need to be addressed to ensure more uniform progress across all schools.

DISCUSSION

In this study, we applied the Variable Returns to Scale (VRS) output-oriented Data Envelopment Analysis (DEA) approach to assess the efficiency of 222 high schools administered by the State of Espírito Santo. The Ideb was considered the primary output, while contextual variables such as the age-grade distortion rate, the socioeconomic level of students, the percentage of white students, and teacher effort were used as inputs. The results revealed significant differences between the reference schools and the other units analyzed, providing important insights for educational policy formulation.

The findings indicate that, in 2017, 17 schools were identified as efficient, serving as benchmarks for other units located in similar contexts but with lower performance. In 2019, this number increased to 18, suggesting a slight improvement in the overall efficiency of the schools over time.

The literature has emphasized that efficiency is not only a function of the resources available but also of the management capacity and quality of teaching offered, especially in adverse contexts (Komalasari et al., 2020; Ainscow, 2020; Cheng, 2022; Cox and Mullen, 2023). This aspect is supported by the contextual indicators observed in the reference schools, which, despite higher age-grade distortion rates, lower socioeconomic levels, and a lower percentage of white students, achieve superior results in the Ideb.

This suggests that the effectiveness of educational policies significantly depends on the schools' ability to adapt strategies to their specific realities, as Cox and Mullen (2023) noted. An exemplary case identified in this study is that of EEEM Gomes Cardim, which, as indicated in Table 2, despite having an Ideb below the average of reference schools, exceeds expectations given its highly vulnerable conditions.

It is important to note that many current public policies tend to underestimate the impact of context on school performance, focusing exclusively on performance indicators without considering the conditions under which schools operate. By emphasizing the significance of contextual variables, this study contributes to a deeper understanding of how these factors influence school efficiency. This approach suggests that future research should adopt a more contextualized perspective, as advocated by various scholars in the educational field (Paschoalotto et al., 2020; Creemers et al., 2022; Dias and Zouain, 2023).

Moreover, identifying reference schools and understanding the practices that lead to greater efficiency, even in adverse contexts, provide valuable insights for replicating these strategies in other educational institutions. This may include developing training programs for school managers focused on optimizing the use of limited resources and implementing strategies to engage and motivate teachers, even in challenging environments, as previously indicated by Creemers et al. (2022). In the context of this study, reference schools were identified in 15 of the 78 municipalities in Espírito Santo. Serra, Colatina, and Muniz Freire stand out for hosting two reference schools each. According to the geographical division proposed by the Brazilian Institute of Geography and Statistics (IBGE), three of these schools are located in the North Coast Region, three in the Northwest Region, seven in the South Region, and five in the Central Region of Espírito Santo, with four belonging to the Metropolitan Region of Greater Vitória.

Regarding the Regional Education Superintendencies (SREs), reference schools were identified in 9 of the 11 SREs (81.820%). All mesoregions of the state have at least one reference school, and about 60% of the analyzed schools have at least one partner school managed by the same SRE. This can facilitate the creation of spaces dedicated to sharing experiences, which are essential for driving educational progress. Figure 6 provides a more detailed visualization of the location of these schools.

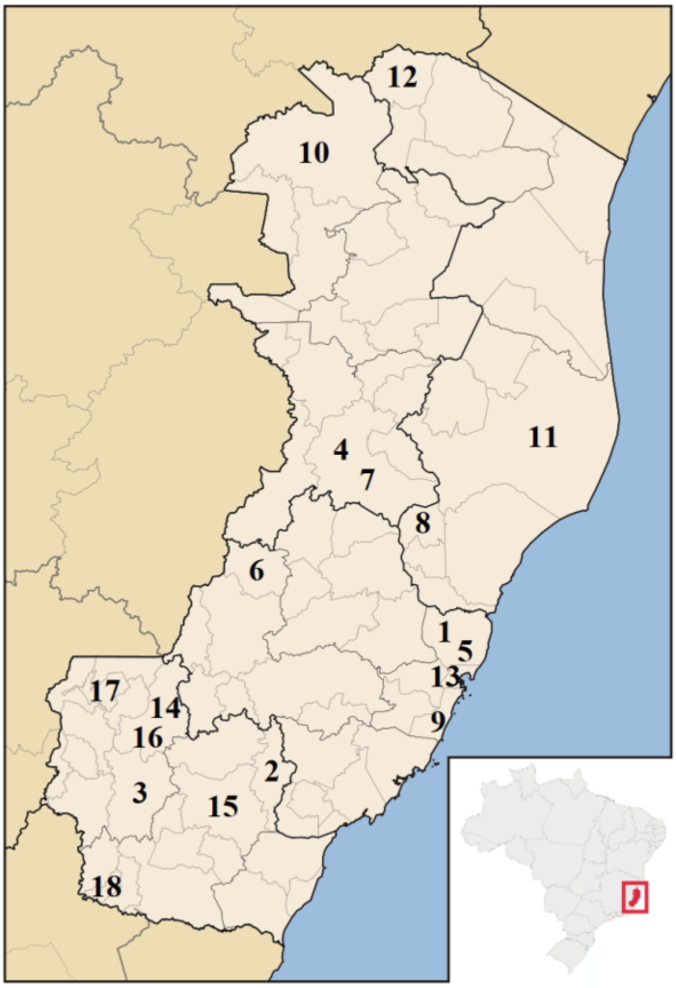

Figure 6: Location of reference schools (own elaboration)

Nonetheless, it is important to acknowledge that DEA assumes all units can learn from and adopt the best practices of reference schools, a premise that may not always be realistic or applicable in practice. Therefore, it is essential to carefully evaluate the feasibility of these best practices and adapt them to each school's unique needs and constraints. This flexible and adaptive approach is critical to ensuring that strategies aimed at improving educational quality are both effective and relevant.

This study also opens avenues for further research. Future investigations could benefit from applying mixed methods and integrating DEA with qualitative approaches such as interviews and case studies to better understand the complexities that influence school performance in challenging environments. Additionally, employing more comprehensive longitudinal data could reveal long-term trends and assess the sustained impact of specific interventions over time.

Limitations of the Analysis

It must be acknowledged that the study has certain limitations. The analysis was based on data from only two cycles of the Ideb, which may not fully capture the long-term dynamics of school performance. Future studies could expand this analysis to include additional Ideb cycles and consider other contextual factors that might influence school efficiency, such as local education policies and specific government interventions. Furthermore, incorporating qualitative methods to complement the quantitative DEA analysis could provide a deeper understanding of the practices and strategies that make some schools more efficient than others. This type of integrated approach could help formulate more effective policies tailored to the realities of Brazilian schools, contributing to ongoing improvements in teaching quality and educational equity.

CONCLUSIONS

Through the application of Data Envelopment Analysis (DEA) to a group of schools located in the State of Espírito Santo, Brazil, this study aimed to identify those operating in challenging contexts that stand out as reference schools when compared to others in similar conditions. The schools identified, spread across various regions of the state, demonstrate that school efficiency is not solely determined by the availability of resources but also by management capacity and adaptability to local conditions.

This study underscores the need for educational policies that recognize and value regional specificities and suggests the creation of spaces for sharing best practices and dialogue between reference schools and their partner schools. By fostering closer collaboration between these schools, it is possible to not only elevate the level of efficiency throughout the educational system but also to promote greater educational equity. Such initiatives, combined with continuous support tailored to local needs, have the potential to transform school realities and sustainably improve educational outcomes. Thus, this study contributes to the discussion on how more contextualized and collaborative educational policies can play a crucial role in enhancing the quality of education in Brazil.

REFERENCES

Ainscow, M. (2020) 'Inclusion and equity in education: Making sense of global challenges', *Prospects*, Vol. 49, No. 3, pp. 123–134. https://doi.org/10.1007/s11125-020-09506-w

Akkari, A. (2021) 'Improving Basic Education in Brazil', *Third International Handbook of Globalisation, Education and Policy Research*, pp. 599–608. https://doi.org/10.1007/978-3-030-66003-1_34

Avilés-Sacoto, E. C., Avilés-Sacoto, S. V., Güemes-Castorena, D. and Cook, W. D. (2021) 'Environmental performance evaluation: A state-level DEA analysis', *Socio-economic Planning Sciences*, Vol. 28, 101082. https://doi.org/10.1016/j.seps.2021.101082

Brasil (1988) Constitution of the Federative Republic of Brazil, DF: Presidência da República: Diário Oficial da União. [Online], Available: http://www.planalto.gov.br/ccivil_03/constituicao/constituicao.htm [9 Aug 2024].

Brasil (2005) Portaria n° 931, de 21 de março de 2005. Establishes the Basic Education Assessment System (Saeb), which will consist of two evaluation processes: the National Basic Education Assessment (Aneb) and the National School Performance Assessment (Anresc), DF: Presidência da República: Diário Oficial da União. [Online], Available: https://download.inep.gov.br/educacao_basica/prova_brasil_saeb/legislacao/Portaria931_NovoSaeb.pdf [9 Aug 2024].

Brasil (2007) Provides for the implementation of the Education Commitment Goals Plan by the Federal Union, DF: Presidência da República: Diário Oficial da União. [Online], Available: https://www.planalto.gov.br/ccivil_03/_ato2007-2010/2007/decreto/d6094.htm [9 Aug 2024].

Brasil (2021) Education Overview: Highlights of Education at a Glance 2021, DF: Instituto Nacional de Estudos e Pesquisas Educacionais Anísio Teixeira. [Online], Available: https://download.inep.gov.br/publicacoes/institucionais/estatisticas_e_indicadores/panorama_da_educacao_destaques_do_education_at_glance_2021.pdf [9 Aug 2024].

Brasil (2024) IDEB 2023 Informative Note, DF: Instituto Nacional de Estudos e Pesquisas Educacionais Anísio Teixeira. [Online], Available: https://download.inep.gov.br/ideb/nota_informativa_ideb_2023.pdf [9 Aug 2024].

Brown, G. T. (2022) 'The past, present and future of educational assessment: A transdisciplinary perspective', *Frontiers in Education*, Vol. 7, p. 1060633. https://doi.org/10.3389/feduc.2022.1060633

Carvalhaes, F., Senkevics, A. S. and Ribeiro, C. A. C. (2023) 'The intersection of family income, race, and academic performance in access to higher education in Brazil', *Higher Education*, Vol. 86, No. 3, pp. 591–616. https://doi.org/10.1007/s10734-022-00916-7

Charnes, A., Cooper, W. W. and Rhodes, E. (1978) 'Measuring the efficiency of decision-making units', *European Journal of Operational Research*, Vol. 2, No. 6, pp. 429–444. https://doi.org/10.1016/0377-2217(78)90138-8

Chen, Y., Ma, X., Yan, P., and Wang, M. (2021) 'Operating efficiency in Chinese universities: An extended two-stage network DEA approach', *Journal of Management Science and Engineering*, Vol. 6, No. 4, pp. 482–498. https://doi.org/10.1016/j.jmse.2021.08.005

Cheng, Y. C. (2022) *School effectiveness and school-based management: A mechanism for development*, 2nd Edition, London: Routledge. https://doi.org/10.4324/9781003267980

Cox, J. S. and Mullen, C. A. (2023) 'Impacting student achievement: Principals' instructional leadership practice in two Title I rural schools', *Journal of School Leadership*, Vol. 33, No. 1, pp. 3–25. https://doi.org/10.1177/10526846221133996

Creemers, B. P., Peters, T. and Reynolds, D. (Eds.) (2022) '*School effectiveness and school improvement*', London: Routledge.

De La Hoz, E., Zuluaga, R. and Mendoza, A. (2021) 'Assessing and Classification of Academic Efficiency in Engineering Teaching Programs', *Journal on Efficiency and Responsibility in Education and Science*, Vol. 14, No. 1, pp. 41–52. https://doi.org/10.7160/eriesj.2021.140104

Delprato, M. and Antequera, G. (2021) 'Public and private school efficiency and equity in Latin America: New evidence based on PISA for development', *International Journal of Educational Development*, Vol. 84, p. 102404. https://doi.org/10.1016/j.ijedudev.2021.102404

Dias, B. F. B. and Zouain, D. M. (2023) 'The "School Effect": A historical debate in defense of an optimistic view of education', *Cadernos de História da Educação*, Vol. 22, p. e187. https://doi.org/10.14393/che-v22-2023-187

Dincă, M. S., Dincă, G., Andronic, M. L. and Pasztori, A. M. (2021) 'Assessment of the European Union's educational efficiency', *Sustainability*, Vol 13, No. 6, p. 3116, https://doi.org/10.3390/su13063116

Doğan, H. (2023) 'Measuring the Efficiency of Turkish Research Universities via Two-Stage Network DEA with Shared Inputs Model', *Journal on Efficiency and Responsibility in Education and Science*, Vol. 16, No. 4, pp. 329–342. https://doi.org/10.7160/eriesj.2023.160406

Ehren, M. (2021) 'International large-scale assessments and education system reform: on the power of numbers', *International Handbook of Comparative Large-Scale Studies in Education: Perspectives, Methods and Findings*, pp. 97–118. https://doi.org/10.1007/978-3-030-88178-8_36

Ernica, M., Rodrigues, E. C. and Soares, J. F. (2024) 'Education Inequalities in Contemporary Brazil: Definition, Measurement, and Outcomes', *Dados*, Vol. 68, No. 1, p. e20220109. https://doi.org/10.1590/dados.2025.68.1.345x

Er-Rays, Y., and M'dioud, M. (2024). 'Assessment of Technical Efficiency in the Moroccan Public Hospital Network: Using the DEA Method', *medRxiv*, Vol. 22, No. 2, pp. 1–15. https://doi.org/10.1101/2024.02.22.24303214

Fernandes, R. and Gremaud, A. P. (2020) 'Large-scale educational assessment and accountability: A brief analysis of the Brazilian experience', *Cadernos de História da Educação*, Vol. 22, e187. https://doi.org/10.14393/che-v22-2023-187

Ferreira, C.M. de C. and Gomes, A.P. (2020) Introduction to Data Envelopment Analysis: theory, models and applications, 2nd Edition, Viçosa: Editora UFV.

Flegl, M., Avilés-Sacoto, S. V., Güemes-Castorena, D. and Avilés-Sacoto, E. C. (2023) 'A State-Level Analysis of Mexican Education and Its Impact on Regional, Economic, and Social Development: Two-Stage Network DEA Approach', *Journal on Efficiency and Responsibility in Education and Science*, Vol. 16, No. 4, pp. 275–286. https://doi.org/10.7160/eriesj.2023.160402

Gangopadhyay, A. (2020) 'Gender, Education, and Programma Bolsa Familia in Brazil', in The Political Economy of the BRICS Countries, pp. 197–216, Singapur: World Scientific. https://doi.org/10.1142/9789811202292_0009

Ghimire, S., Amin, S. H. and Wardley, L. J. (2021) 'Developing new data envelopment analysis models to evaluate the efficiency in Ontario Universities', *Journal of Informetrics*, Vol. 15, No. 3, p. 101172. https://doi.org/10.1016/j.joi.2021.101172

Gökgöz, F. and Yalçın, E. (2022) 'A slack-based DEA analysis for the world cup teams', *Team Performance Management: An International Journal*, Vol. 28, No. 1/2, pp. 1–20. http://dx.doi.org/10.1108/TPM-07-2021-0050 Parte superior do formulárioParte inferior do formulário

Gomes, F.P. (1985) '*Curso de estatística experimental*', [Experimental statistics course]. Piracicaba: ESALQ/USP.

Guzmán-Raja, I. and Guzmán-Raja, M. (2021) 'Measuring the efficiency of football clubs using data envelopment analysis: Empirical evidence from Spanish professional football', *Sage Open*, Vol. 11, No. 1. https://doi.org/10.1177/2158244021989257

Hajiagha, S. H. R., Amoozad Mahdiraji, H., Hashemi, S. S., Garza-Reyes, J. A. and Joshi, R. (2023) 'Public hospitals performance measurement through a three-staged data envelopment analysis approach: evidence from an emerging economy', *Cybernetics and Systems*, Vol. 54, No. 1, pp. 1–26. https://doi.org/10.1080/01969722.2022.2055382

Halásková, R., Mikušová Meričková, B. and Halásková, M. (2022) 'Efficiency of Public and Private Service Delivery: The Case of Secondary Education', *Journal on Efficiency and Responsibility in Education and Science*, Vol. 15, No. 1, pp. 33–46. https://doi.org/10.7160/eriesj.2022.150104

Halásková, M., Gavurova, B. and Kocisova, K. (2020) 'Research and development efficiency in public and private sectors: An empirical analysis of EU countries by using DEA methodology', *Sustainability*, Vol. 12, No. 17, p. 7050. https://doi.org/10.3390/su12177050

Hančlová, J. and Chytilová, L. (2023). 'Education Performance of Czech Public Higher Education Institutions Using Data Envelopment and Panel Regression Analysis', *Journal on Efficiency and Responsibility in Education and Science*, Vol. 16, No. 4, pp. 313–328. https://doi.org/10.7160/eriesj.2023.160405

Henry, D. A., Cortés, L. B. and Votruba-Drzal, E. (2020) 'Black–White achievement gaps differ by family socioeconomic status from early childhood through early adolescence', *Journal of Educational Psychology*, Vol. 112, No. 8, pp. 1471–1489. https://doi.org/10.1037/edu0000439

Jerrim, J. and Sims, S. (2022) 'School accountability and teacher stress: international evidence from the OECD TALIS study', *Educational Assessment, Evaluation and Accountability*, Vol. 34, No. 1, pp. 5–32. https://doi.org/10.1007/s11092-021-09360-0

Jiang, J., Lee, S. K. and Rah, M. J. (2020) 'Assessing the research efficiency of Chinese higher education institutions by data envelopment analysis', *Asia Pacific Education Review*, Vol. 21, No. 3, pp. 423–440. https://doi.org/10.1007/s12564-020-09634-0

Jimenez, L. and Modaffari, J. (2021) 'Effective and Equitable Assessment Systems', in *Future of Testing in Education*. Center for American Progress. Available: https://eric.ed.gov/?id=ED617053 [9 Aug 2024].

Johnes, J. (2006) 'Data envelopment analysis and its application to the measurement of efficiency in higher education', *Economics of Education Review*, Vol. 25, No. 3, pp. 273–288. https://doi.org/10.1016/j.econedurev.2005.02.005

Komalasari, K., Arafat, Y. and Mulyadi, M. (2020) 'Principal's management competencies in improving the quality of education', *Journal of Social Work and Science Education*, Vol. 1, No. 2, pp. 181–193. https://doi.org/10.52690/jswse.v1i2.47

Koronakos, G. (2019) 'A taxonomy and review of the network data envelopment analysis literature', *Machine Learning Paradigms: Applications of Learning and Analytics in Intelligent Systems*, pp. 255–311. https://doi.org/10.1007/978-3-030-15628-2_9

Li, Z., Feng, C. and Tang, Y. (2022) 'Bank efficiency and failure prediction: a nonparametric and dynamic model based on data envelopment analysis', *Annals of Operations Research*, Vol. 315, No. 1, pp. 279–315. https://doi.org/10.1007/s10479-022-04597-4

Lima, L.C.A. (2011) 'Pesquisa em eficácia escolar: origem e trajetórias', *Estudos em Avaliação Educacional*, Vol. 22, No. 50, p. 593. https://doi.org/10.18222/eae225020111972

Liu, J. S., Lu, L. Y., Lu, W. M. and Lin, B. J. (2013) 'Data envelopment analysis 1978–2010: A citation-based literature survey', *Omega*, Vol. 41, No. 1, pp. 3–15. https://doi.org/10.1016/j.omega.2010.12.006

Liu, J., Peng, P., and Luo, L. (2020) 'The Relation between Family Socioeconomic Status and Academic Achievement in China: A Meta-analysis', *Educational Psychology Review*, Vol. 32, No. 1, pp. 49–76. https://doi.org/10.1007/s10648-019-09494-0

Mammadov, R. and Aypay, A. (2020) 'Efficiency analysis of research universities in Turkey', *International Journal of Educational Development*, Vol. 75, p. 102176. https://doi.org/10.1016/j.ijedudev.2020.102176

Moncayo–Martínez, L. A., Ramírez–Nafarrate, A. and Hernández–Balderrama, M. G. (2020) 'Evaluation of public HEI on teaching, research, and knowledge dissemination by Data Envelopment Analysis', *Socio-Economic Planning Sciences*, Vol. 69, p. 100718. https://doi.org/10.1016/j.seps.2019.06.003

Moraes, C. P., Peres, R. T. and Pedreira, C. E. (2021) 'Eficácia escolar e variáveis familiares em tempos de pandemia: um estudo a partir de dados do ENEM', *Interfaces da Educação*, Vol. 12, No. 35, pp. 635–658. https://doi.org/10.26514/inter.v12i35.5785

Oliveira, E. S., Justo, W. R. and De Lucena, M. A. (2024) 'The Dynamics Of Public High School Student Performance In Ceará: A Study Of The Case Of Sobral In Brazil', *Journal Of Humanities And Social Science*, Vol. 29, No. 3, pp. 12–24. https://doi.org/10.9790/0837-2903011224

Organisation for Economic Co-operation and Development (OECD) (2023). *PISA 2022 Results: South America Overview*, [Online], Available: https://www.oecd.org/en/publications/pisa-2022-results-volume-i_53f23881-en.html [24 Aug 2024].

Özsoy, V. S. and Örkcü, H. H. (2021) 'Structural and operational management of Turkish airports: a bootstrap data envelopment analysis of efficiency', *Utilities Policy*, Vol. 69, 101180. https://doi.org/10.1016/j.jup.2021.101180

Paschoalotto, M. A. C., Passador, J. L., Passador, C. S. and Oliveira, P. H. D. (2020) 'Local government performance: Evaluating efficiency, efficacy, and effectiveness at the basic education level', *BAR-Brazilian Administration Review*, Vol. 17, No. 4, e190149. https://doi.org/10.1590/1807-7692bar2020190149

Pereira, M. A., Ferreira, D. C., Figueira, J. R. and Marques, R. C. (2021) 'Measuring the efficiency of the Portuguese public hospitals: A value modelled network data envelopment analysis with simulation', *Expert Systems with Applications*, Vol. 181, 115169. https://doi.org/10.1016/j.eswa.2021.115169

Popović, M., Savić, G., Kuzmanović, M. and Martić, M. (2020) 'Using data envelopment analysis and multi-criteria decision-making methods to evaluate teacher performance in higher education', *Symmetry*, Vol. 12, No. 4, 563. https://doi.org/10.3390/sym12040563

Queiroga, E. M., Siqueira, E. S., Portela, C. S., Cordeiro, T. D., Bittencourt, I. I., Isotani, S., Mello, R. F., Muñoz, R., and Cechinel, C. (2024) 'Data-Driven Strategies for Achieving School Equity: Insights From Brazil and Policy Recommendations', *IEEE Access*, Vol. 12, pp. 101646–101659. https://doi.org/10.1109/ACCESS.2024.3422822

R Core Team (2023). '*R: A language and environment for statistical computing*', Vienna: R Foundation for Statistical Computing. Available: https://www.r-project.org/

Ramzi, S., Afonso, A. and Ayadi, M. (2016) 'Assessment of efficiency in basic and secondary education in Tunisia: A regional analysis', *International Journal of Educational Development*, Vol. 51, pp. 62–76. https://doi.org/10.1016/j.ijedudev.2016.08.003

Ranieri, N. (2024) 'Education Federalism in Brazil: Contradictions, Challenges, and Possibilities', in *A Comparative Analysis of Systems of Education Law*, Singapore: Springer Nature Singapore, pp. 53–62. https://doi.org/10.1007/978-981-97-1052-2_3

Richter, L. M. and Vieira, M. S. (2021) 'O Ideb e seus efeitos na organização do trabalho escolar', *Revista Brasileira de Política e Administração da Educação - Periódico científico editado pela ANPAE*, Vol. 37, No. 1, pp. 327–350. https://doi.org/10.21573/vol37n12021.105962

Rutkowski, D., Thompson, G. and Rutkowski, L. (2020) 'Understanding the Policy Influence of International Large-Scale Assessments in Education',in Wagemaker, H. (ed.) *Reliability and Validity of International Large-Scale Assessment*, Cham. Switzerland: Springer. https://doi.org/10.1007/978-3-030-53081-5_15

Ryan, A., Barchers, C., Christofa, E. and Knodler, M. (2021) 'Equitable resource allocation for municipal safety: A data envelopment analysis', *Transportation Research Part D: Transport and Environment*, Vol. 97, 102926. http://dx.doi.org/10.1016/j.trd.2021.102926

Salas-Velasco, M. (2020) 'The technical efficiency performance of the higher education systems based on data envelopment analysis with an illustration for the Spanish case', *Educational Research for Policy and Practice*, Vol. 19, No. 2, pp. 159–180. https://doi.org/10.1007/s10671-019-09254-5

Santos Tavares, R., Angulo-Meza, L. and Parracho Sant'Anna, A. (2021) 'A proposed multi-stage evaluation approach for Higher Education Institutions based on network Data envelopment analysis: A Brazilian experience', *Evaluation and Program Planning*, Vol. 89, 101984. https://doi.org/10.1016/j.evalprogplan.2021.101984

Silva, M. C., Camanho, A. S. and Barbosa, F. (2020) 'Benchmarking of secondary schools based on Students' results in higher education', *Omega*, Vol. 95, 102119. https://doi.org/10.1016/j.omega.2019.102119

Silva, M. C. (2021) '*School effectiveness and equity in a philanthropic educational institution*', Doctoral thesis, Pontifical Catholic University of Rio de Janeiro, [Online], Available: https://www.maxwell.vrac.puc-rio.br/53671/53671.PDF [20 Aug 2024].

Sinuany-Stern, Z. and Hirsh, A. (2021) 'The relative efficiencies of higher education in OECD countries', *Handbook of Operations Research and Management Science in Higher Education*, pp. 481–512. https://doi.org/10.1007/978-3-030-74051-1_16

Soares, D. J. M., Soares, T. E. and Santos, W. (2023) 'Approaches between No Child Left Behind and Ideb/Brasil: contributions to the debate', *Revista Ibero-Americana De Estudos Em Educação*, Vol. 18, e023005. https://doi.org/10.21723/riaee.v18i00.16848

Soares, J.F. (2016) 'O direito à educação no contexto da avaliação educacional', *Em Aberto*, Vol. 29, No. 96, pp. 141–152.

Soares, J. F. and Xavier, F. P. (2013) 'Pressupostos educacionais e estatísticos do Ideb', *Educação & Sociedade*, Vol. 34, No. 124, pp. 903–923. https://doi.org/10.1590/s0101-73302013000300013

Soares, D. J. M., Soares, T. E. A. and Santos, W. (2022) 'Contributions of data envelopment analysis in studies on school effectiveness', *Estudos Em Avaliação Educacional*, Vol. 33, e09103. https://doi.org/10.18222/eae.v33.9103

Soares, D. J. M. and Santos, W. (2024) 'Indicadores de avaliação de contexto e resultados educacionais no Ideb: uma análise das escolas estaduais de ensino médio no Espírito Santo', *Revista Brasileira de Estudos Pedagógicos*, Vol. 105, No. 2, e5872.

Torres-Samuel, M., Vásquez, C. L., Luna, M., Bucci, N., Viloria, A., Crissien, T. and Manosalva, J. (2020) 'Performance of education and research in Latin American countries through data envelopment analysis (DEA)', *Procedia Computer Science*, Vol. 170, pp. 1023–1028. https://doi.org/10.1016/j.procs.2020.03.079

Ullah, S., Majeed, A. and Popp, J. (2023) 'Determinants of bank's efficiency in an emerging economy: A data envelopment analysis approach', *Plos One*, Vol. 18, No. 3, e0281663. https://doi.org/10.1371/journal.pone.0281663

Vermunt, J. D., Vrikki, M., Dudley, P. and Warwick, P. (2023) 'Relations between teacher learning patterns, personal and contextual factors, and learning outcomes in the context of Lesson Study', *Teaching and Teacher Education*, Vol. 133, 104295. https://doi.org/10.1016/j.tate.2023.104295

Wyszynski, A. (2024) 'Efficiency of Professional Polish Volleyball Clubs PlusLiga, Poland: Data Envelopment Analysis Approach', *Journal of Eastern Europe Research in Business and Economics*, pp. 1–16. https://doi.org/10.5171/2024.746883

Xu, J., Jiang, Y., Guo, X., and Jiang, L. (2021) 'Environmental efficiency assessment of heavy pollution industry by data envelopment analysis and malmquist index analysis: empirical evidence from China', *International Journal of Environmental Research and Public Health*, Vol. 18, No. 11, pp. 5761–5781. https://doi.org/10.3390/ijerph18115761

Yin, X., Wang, J., Li, Y., Feng, Z. and Wang, Q. (2021) 'Are small towns really inefficient? A data envelopment analysis of sampled towns in Jiangsu province, China', *Land Use Policy*, Vol. 109, 105590. https://doi.org/10.1016/j.landusepol.2021.105590

Zeng, Z., Yang, W., Zhang, S. and Witlox, F. (2020) 'Analysing airport efficiency in East China using a three-stage data envelopment analysis', *Transport*, Vol. 35, No. 3, pp. 255–272. https://doi.org/10.3846/transport.2020.12869

Zubir, M. Z., Noor, A. A., Mohd Rizal, A. M., Harith, A. A., Abas, M. I., Zakaria, Z. and A. Bakar, A. F. (2024) 'Approach in inputs & outputs selection of Data Envelopment Analysis (DEA) efficiency measurement in hospitals: A systematic review', *Plos One*, Vol. 19, No. 8, e0293694. https://doi.org/10.1371/journal.pone.0293694

Zuluaga, R., Camelo-Guarín, A. and De La Hoz, E. (2023) 'Assessing the Relative Impact of Colombian Higher Education Institutions Using Fuzzy Data Envelopment Analysis (Fuzzy-DEA) in State Evaluations', *Journal on Efficiency and Responsibility in Education and Science*, Vol. 16, No. 4, pp. 299–312. https://doi.org/10.7160/eriesj.2023.160404