RELATIONS BETWEEN SCIENTIFIC REASONING, CULTURE OF PROBLEM SOLVING AND PUPIL´S SCHOOL PERFORMANCE

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Highlights

- *The ability to use the existing knowledge correlates strongly to the three dimensions of Scientific reasoning (proportional reasoning, control of variables and probability reasoning)*
- *It wasn't proved any correlation between the creativity and the dimensions of Scientific reasoning*
- *Individual components of the Culture of problem solving and individual dimensions of Scientific reasoning largely do not correlate with school performance either in mathematics or in physics*

Abstract

The article reports the results of a study, the main aim of which was to find out correlations among the three components of the Culture of problem solving (reading comprehension, creativity and ability to use the existing knowledge) and six dimensions of Scientific reasoning (conservation of matter and volume, proportional reasoning, control of variables, probability reasoning, correlation reasoning and hypothetical-deductive reasoning). Further, we present the correlations among individual components of the Culture of problem solving and individual dimensions of Scientific reasoning with pupils' school performance in mathematics and physics. We conducted our survey among 23 pupils aged between 14–15 years in the Ústí nad Labem Region. The results have shown that one component of the Culture of problem solving – the ability to use the existing knowledge – strongly correlates with three dimensions of the Scientific reasoning structure: proportional reasoning, control of variables and probability reasoning. However, no correlation was proved between the creativity and the dimensions of Scientific reasoning. We have found out also that the indicators of the Culture of problem solving and the Scientific reasoning largely do not correlate with school performance either in mathematics or in physics.

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Introduction

The paper is one of the outcomes of the research project concerning with developing culture of solving problems in school mathematics. It deals with mutual relations between the Culture of problem solving (CPS) and Scientific reasoning (SR). The paper is an extension of the contribution (Eisenmann et al., 2017), which was presented at the conference ERIE 2017. This article reports more detailed the results of a study, the aim of which was to find out correlations among the three components of the Culture of problem solving (reading comprehension, creativity and ability to use the existing knowledge) and six dimensions of Scientific reasoning (conservation of matter and volume, proportional reasoning, control of variables, probability reasoning, correlation reasoning and hypothetical-deductive reasoning).

The constructs CPS and SR aim at assessment of pupils' abilities to solve problems, which we believe are significantly developed in mathematics and physics. Therefore we explore also correlations among pupil's school performance in these subjects and both constructs that we present also in this paper. In the section Results and discussion (see Tables 1, 2, 3, 4 and 5) we bring out the description of correlations among all pairs of examined components of CPS and dimensions of SR and report also correlations of these indicators of CPS and SR with school performance in mathematics and physics. In more detail we comment possible reasons of stronger or weaker relations among them.

The Culture of problem solving

In our research, we have been engaged in the area of problem solving for a long time (Novotná et al., 2014; Novotná, Eisenmann and Přibyl, 2015). In order to be able to describe a pupil's ability to solve mathematical problems, we have introduced the so-called Culture of problem solving construct within the research mentioned above. The phrase 'culture of problem solving' can be found in several pieces of work (e.g. Clarke, Goos and Morony; 2007; Reiss and Törner, 2007), where the word culture is not strictly defined and can be understood as a more cultivated approach to the studied phenomenon. Such authors as Clarke, Goos and Morony (2007) link the word culture to the word inquiry – culture of inquiry. When forming the phrase CPS, the word culture was understood from us as a system of various meanings, activities and patterns of behaviour that can be met with in problem solving at schools.

When composing the components of the structure, we drew on previous works (e.g. Herl et al., 1999; Schoenfeld, 1982; Szetela, 1987; Szetela and Nicol, 1992, Wu and Adams, 2006), among which the work of Wu and Adams (2006) was the most relevant. Their problem-solving profile, conceived as a tool for changing a pupil's ability to solve problems, focuses on two components of the structure we developed: Reading/Extracting all information from the question and Mathematics concepts, mathematization and reasoning.

The composition of CPS is described in detail in (Eisenmann et al., 2015). This structure consists of four components: intelligence, reading comprehension, creativity and ability to use the existing knowledge.

There are no doubts about the indispensability of including *intelligence* (I) in the structure of CPS. As Wenke, Frensch and Funke (2005) state, from the inception of the concept of 'intelligence', the ability to solve problems has featured prominently in virtually every definition of human intelligence. In addition, intelligence has often been viewed as one of the best predictors of the problem-solving ability.

The second component is *reading comprehension* (RC). Obviously, this is one of the key competences without which successful problem solving would be impossible, as pointed out by a number of authors (Pape, 2004; Schoenfeld, 1992) and verified by Hite (2009). The inclusion of this component is based on Pólya's four stages of solving a problem (Pólya, 2004). The first stage is understanding the problem. The basis of solving any problem is to understand its structure connected with the ability to read the assignment of the problem with comprehension. This means that having read the assignment, the solver is able to grasp the relations in the problem, identify the initial and output variables of the problem and handle the input data in an appropriate way.

The third component is *creativity* (C). The key role of creativity in problem solving is discussed by Bahar and Maker (2011) or Sriraman (2005). Nadjafikhah, Yaftian and Bakhshalizadeh (2012) speak of creative problem solving. "At the school level, creativity in mathematics is generally related to problem solving and or problem posing." (Nadjafikhah, Yaftian and Bakhshalizadeh, 2012: 290). Chamberlin and Moon (2005: 38) state that "Creativity refers to the domain-specific thinking processes used by mathematicians when engaged in non-routine problem solving."

The fourth component is the *ability to use the existing knowledge* (UK). This ability has been considered as a prerequisite to successful solving of non-routine problems. Whilst solving such kinds of problems, the knowledge itself is not sufficient; the solver must also be able to use it.

With respect to an individual pupil, we find the use of CPS in teaching important in three areas.

1. Knowing pupil's CPS may help the teacher select appropriate problems the pupil will be able to solve successfully.
2. It may help eliminate a pupil's weaknesses that may be an obstacle to solving problems.
3. Knowing pupil's CPS may help the teacher decide which heuristic strategies should be used and in being aware of the depth in which these strategies can be handed over to the pupil.

Scientific reasoning

Research on scientific reasoning has its roots in the early studies on cognitive development of 'formal reasoning' (Inhelder and Piaget, 1958; Piaget, 1965) and 'critical thinking' (Hawkins and Pea, 1987). What exactly constitutes scientific reasoning is complex issue, therefore there are many definitions of scientific reasoning. Lawson (1982, 2005) suggests that scientific reasoning has a structure that follows from hypothetic-deductive nature of science that includes such aspects as proportional reasoning, control of variables, probability reasoning, correlation reasoning and process of drawing inferences from initial premises, which is linked with inductive and deductive reasoning. Scientific reasoning involves application of the methods of scientific inquiry to reasoning or problem-solving situations, for example systematically exploring a problem, formulating and testing hypotheses and evaluating experimental outcomes. According Opitz, Heene and Fischer (2017: 81), the differences between conceptualizations of scientific reasoning that exist are in:

1. the skills they include,
2. if there is a general, uniform scientific reasoning ability or rather more differentiated dimensions of scientific reasoning, and
3. if they assume scientific reasoning to be domain general or domain specific.

With regard to our research, in the next text we will aim specifically at measuring of scientific reasoning and therefore we will deal with the operational definition of scientific reasoning. It includes the necessary skills that support scientific inquiry such as control of variables, hypothetical-deductive reasoning, causal and correlational reasoning, proportions and ratios deductive and inductive reasoning, and probabilistic reasoning (Han, 2013). This is not a complete list because scientific reasoning is multifaceted and other skills could be included but these ones are commonly agreed-upon skills that are needed for students to conduct scientific inquiry.

For assessment of scientific reasoning the following dimensions can be used (Han, 2013):

- Control of Variables
- Proportions and Ratios
- Probability
- Correlational Reasoning
- Deductive Reasoning
- Inductive reasoning
- Causal Reasoning
- Hypothetical-Deductive Reasoning

In our study we used the Lawson Classroom Test of Scientific Reasoning (LCTSR) (Lawson, 1978) that was designed to examine six dimensions including conservation of matter and volume, proportional reasoning, control of variables, probability reasoning, correlation reasoning and hypothetical-deductive reasoning. These skills are important components of the broadly defined scientific reasoning ability. In our study we restrict only to these chosen dimensions and scientific reasoning we define operationally in terms of students' ability in handling questions of these six skill dimensions.

Scientific inquiry is considered the core component of STEM education (Science, Technology, Engineering and Math), therefore the scientific reasoning skills are emphasized in science education. The development of these skills, however, cannot be separated from prior knowledge and the learning of content because of their tightly linking, how research have shown for example in physics education (Coletta and Phillips, 2005). Childrens' reasoning skills are interesting not only for researchers but also for teachers who can determine the best methods for improving learning and instruction in science education (Zimmerman, 2007). Some research (e.g. Papáček, 2010) shown that particularly inquiry-based science instruction can promote scientific reasoning abilities. However, creative thinking and inquiry learning can be promoted in any classroom, not only in science.

The relation between CPS and SR

Much research has been carried out to find out how scientific reasoning relates to other areas of learning. For example Shayer and Adey (1993) argue that instruction in scientific reasoning has a permanent impact on general learning ability. They carried out a study comparing students who received scientific reasoning-based teaching with those who did not. They showed that the reasoning-based group (at age of 16) outperformed the control group on tests not only in Science but also in English and Mathematics.

The P21 (Partnership for 21st Century Skills – a group of corporations who partnered with the U.S. Department of Education in 2002) has created a framework that identifies the key skills for success, the so called '21st Century skills'. These include, among others, creativity, critical thinking and problem solving. Scientific reasoning skills are good tools for the purpose of the development of these key skills. As CPS includes reading comprehension and creativity in its structure, we suppose that a relation exists between CPS and SR and this is the point we wish to focus on in our contribution.

Objectives

We endeavour to find potential correlations between individual CPS components and SR dimensions by means of appropriate research with pupils. This contribution describes the first pilot research the aim of which, in particular, is to formulate first hypotheses concerning correlations between individual CPS components and SR dimensions. As it is possible to expect, with high probability, that intelligence (the first CPS component) has a relation to all above-mentioned SR dimensions, we have narrowed our pilot research to the testing of the three remaining CPS components. The secondary task of the pilot research is to carry out the mapping of any possible relations between individual SR dimensions. We explore also correlations among pupil's school performance in mathematics and physics and both constructs that we present in this paper.

Materials and methods

The following subsections focus on the art of measuring both the constructs and the description of the sample.

Culture of problem solving

As far as RC is concerned, the pupils were set a short text of 15 lines. Afterwards, their task was to answer correctly 9 questions (from 4-item multiple-choice possibilities they were selecting one correct answer). The aggregate of all points has formed the total score. The test is built on the same principle as the one used in the PISA research.

In our study, *C* was understood in the context of divergent thinking. In accordance with Sternberg (2005), we do not perceive creativity as a single attribute but a set of attributes, and with respect to the study we selected a set of strategies to focus on. The creativity level was measured by Guilford's Alternative Uses Test, which measures the following four dimensions:

- Fluency – how many relevant uses the pupil proposes;
- Originality – how unusual these uses are;
- Flexibility – how many areas the answers refer to;
- Elaboration – quality and number of details in the answer.

The pupils proposed as many 'uses of common objects' as possible. What is important here is how logical and practicable the answers were. Qualitative evaluation of each dimension was translated into points and the total score indicating an index of creativity. The higher the index, the more creative the pupil is assessed to be.

The pupils' UK was assessed on the basis of a set of problems developed by the research team. Dyads of problems were used for this written testing – the first problem to find out whether a pupil has a particular piece of knowledge and the other to find out whether the pupil can use or apply it. The more frequent is the situation in which the pupil has the required knowledge and can use it at the same time (i.e. both the tasks of the given dyad are solved correctly), the higher is the score he will achieve in the area of this component.

Example of a dyad:

a) Solve the equation: $6x + 4x + 2x = 18$

b) There are three vessels of water. Each of them has a different volume and in total they contain 19.5 litres of water. The largest vessel contains twice as much water as the medium one and the medium vessel contains four times more water than the smallest one. How many litres of water are in each of the vessels?

All three above-named CPS components have been tested during the course of a single 45-minute teaching unit. The section of the test focused on RC lasted 14 minutes, C section was expected to be completed in 7 minutes, and the third section, which concentrated on UK lasted 14 minutes as well. All tested pupils were working independently, they were not allowed to use either calculators or any other technological devices. All parts of the test were then evaluated by the authors of the contribution themselves.

Scientific reasoning

Scientific reasoning was tested by LCTSR (Lawson, 1978) which has gained the largest popularity among researchers and teachers. Since its initial development, the test has undergone several revisions. We used the Czech version (Dvořáková, 2016) of the current version of LCTSR released in 2000 and according to Han (2013) we carried out small corrections in items 8a and 8b.

LCTSR is a 24-item, two-tier test which involve a series of multiple-choice questions. Each of the two-tier items (from the total number of 12 pairs) consists of a question with some possible answers followed by a second question giving possible reasons for the response to the first question. The reasoning options are often based on student misconceptions that were discovered via free response tests and interviews (Driver et al., 2003; Stepans, 2003).

LCTSR assesses students' reasoning abilities in six dimensions including conservation of matter (items 1, 2) and volume (CONSER) (items 3, 4), proportional reasoning (PROPOR) (items 5, 6, 7, 8), control of variables (VARIABL) (items 9, 10, 1, 12, 13, 14), probability reasoning (PROBAB) (items 15, 16, 17, 18), correlation reasoning (CORREL) (items 19, 20) and hypothetical-deductive reasoning (HYPDED) (items 21, 22, 23, 24).

The items have an increasing difficulty. With regard to the evaluation of the test, for tasks 1 through to 22 the points are awarded only when both the related tasks are resolved correctly. Only tasks 23 and 24 are independent and for that reason they are also evaluated independently.

Example of the two-tier item (9 and 10)

9. At the right are drawings (see Figure 1) of three strings hanging from a bar. The three strings have metal weights attached to their ends. String 1 and String 3 are the same length. String 2 is shorter. A 10 unit weight is attached to the end of String 1. A 10 unit weight is also attached to the end of String 2. A 5 unit weight is attached to the end String 3. The strings (and attached weights) can be swung back and forth and the time it takes to make a swing can be timed.

Suppose you want to find out whether the length of the string has an effect on the time it takes to swing back and forth. Which strings would you use to find out?

a. only one string
b. all three strings
c. 2 and 3

d. 1 and 3
e. 1 and 2

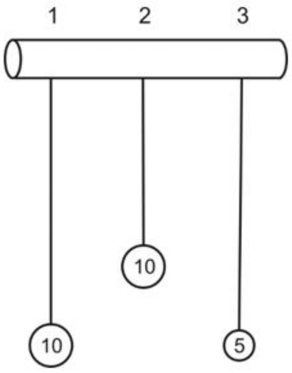

Figure 1: Picture from the item 9 (source: Dvořáková, 2016)

10. because
   a. you must use the longest strings.
   b. you must compare strings with both light and heavy weights.
   c. only the lenghts differ.
   d. to make all possible comparisons.
   e. the weights differ.

In our study we aimed at the ninth grade of primary school because Han (2013) found out that LCTSR worked well just with 9th graders. The pupils were solving the task during one teaching unit. At first they were briefly instructed by their teacher and then they received approx. 30 minutes for the solution of the test. They worked independently and were not allowed to use either calculators or tables. The test was evaluated by the authors of this paper.

Sample

Altogether 23 pupils (12 girls and 11 boys) from one class of the ninth grade aged between 14–15, from one primary school in Teplice took part in our pilot study. Describing the pupils' school performance, they can be characterized as common learners, representing above average class in the Czech Republic. Such evaluation has been backed up by two sources: the grade in mathematics and physics in the 2015/2016 school report and the evaluation of their mathematics and physics teacher. The arithmetic mean of the grade achieved in mathematics is 2.0 with standard deviation of 0.52, in physics 2.1 with standard deviation of 0.54.

Statistical evaluation

On account of type variables we used Spearman's rank correlation coefficients to measure the strength of relationship between two variables. The calculation was realized by STATISTICA 12.0 (StatSoft, Inc.).

Results and discussion

At first we explored correlations between RC, C and UK components of CPS. The results can be seen in Table 1.

Pairs of components	N	Spearman *R*	R^2	*p*-level
RC & UK	20	0.0917	0.0084	0.7003
RC & C	20	0.3324	0.1105	0.1520
UK & C	19	– 0.2472	0.0611	0.3074

Table 1: Spearman correlation coefficients for components of CPS construct (source: own calculation)

With none of the pairs it is possible to reject a null hypothesis that correlation coefficient is zero at the 5% level of significance. The components RC, C and UK are independent. It is surprising that the correlation coefficient between RC and C is positive, whereas between UK and C is negative.

Similarly the correlations between dimensions of SR were explored. The results can be seen in Table 2. Marked correlations are significant at $p < 0.05000$ (bold types in the table).

Pairs of dimensions	N	Spearman *R*	R^2	*p*-level
CONSER & PROPOR	**18**	**0.6509**	**0.4237**	**0.0034**
CONSER & VARIABL	18	0.4086	0.1670	0.0921
CONSER & PROBABL	**18**	**0.5135**	**0.2636**	**0.0292**
CONSER & CORREL	18	– 0.0627	0.0039	0.8045
CONSER & HYPDED	18	0.2970	0.0882	0.2313
PROPOR & VARIABL	**18**	**0.6165**	**0.3801**	**0.0064**
PROPOR & PROBABL	**18**	**0.5426**	**0.2944**	**0.0199**
PROPOR & CORREL	18	0.0119	0.0001	0.9625
PROPOR & HYPDED	18	0.3935	0.1548	0.1061
VARIABL & PROBABL	**18**	**0.5044**	**0.2544**	**0.0327**
VARIABL & CORREL	18	0.3195	0.1020	0.1962
VARIABL & HYPDED	18	0.0890	0.0079	0.7253
PROBABL & CORREL	18	0.2845	0.0809	0.2524
PROBABL & HYPDED	18	0.0433	0.0018	0.8644
CORREL & HYPDED	18	– 0.0112	0.0001	0.9646

Table 2: Spearman correlation coefficients for dimensions of SR (source: own calculation)

Although the sample size is not so extent, we find five pairs of strongly correlated dimensions, in which we can reject the null hypothesis at the 5% level of significance (bold types in the table). Therefore, four dimensions correlate together (CONSER, PROPOR, VARIABL a PROBAB), with the remaining two dimensions (CORREL a HYPDED) no correlation with any further dimension has been revealed.

The main target of our research, however, was to investigate correlations between the SR dimensions and the CPS components. The results can be seen in Table 3.

Pairs of dimensions	N	Spearman *R*	R^2	*p*-level
RC & CONSER	16	0.2222	0.0493	0.4081
RC & PROPOR	16	0.2505	0.0627	0.3492
RC & VARIABL	16	0.4497	0.2022	0.0804
RC & PROBABL	16	0.2139	0.0457	0.4261
RC & CORREL	16	– 0.0139	0.0001	0.9592
RC & HYPDED	16	– 0.4837	0.2340	0.0576
UK & CONSER	15	0.4735	0.2242	0.0745
UK & PROPOR	**15**	**0.8142**	**0.6630**	**0.0002**
UK & VARIABL	**15**	**0.5741**	**0.3296**	**0.0251**
UK & PROBABL	**15**	**0.5413**	**0.293**	**0.0371**
UK & CORREL	15	– 0.1572	0.0247	0.5757
UK & HYPDED	15	0.2048	0.0419	0.4640
C & CONSER	15	0.3527	0.1244	0.1971
C & PROPOR	15	0.1311	0.0171	0.6413
C & VARIABL	15	0.0271	0.0007	0.9233
C & PROBABL	15	0.0611	0.0037	0.8287
C & CORREL	15	0.2050	0.0420	0.4633
C & HYPDED	15	0.1395	0.0194	0.6199

Table 3: Spearman correlation coefficients for components of CPS construct and dimensions of SR (source: own calculation)

Three strongly correlating pairs can be observed here, where we reject the null hypothesis at the 5% level of significance (bold types in the table). With the UK component of CPS three dimensions of SR correlate like this – PROPOR, VARIABL a PROBAB. In addition, three other pairs have been found which correlate in a weaker manner; here the null hypothesis can be rejected at the 10% level of significance.

The correlation coefficients show that there is a narrow relationship between the first four dimensions of SR (excluding CONSER and VARIABL). The understanding of the conservation of mass and volume (CONSER) and proportional reasoning (PROPOR) are basic skills that children usually develop at a fairly young age. The abilities of the control of variables (VARIABL) and understanding probability (PROBAB) start to develop more significantly at the end of primary school and the process continues throughout the first years of secondary school (Han, 2013). Individual dimensions of SR are not independent, but they create a hierarchy, which means that the successful solution of tasks from the higher dimension supposes the mastering of tasks from the lower dimensions. Based on the discovery of strong correlations between the first four dimensions, we assume that their good mastering creates the necessary background for the successful development of reasoning skills in the area of higher dimensions for this age category (at the end of primary school).

In our research we have noticed only weak correlations between CORREL and the previous four dimensions, which are not, however, statistically significant. This finding could be connected with the fact that only a one pair-item is devoted to this dimension; thus pupils can get either two points or no point at all.

The hypothetical-deductive reasoning (HYPDED) is the most complicated ability in the LCTSR, which represents the last stage of formal reasoning. This dimension is developed in particular during the students' stay at secondary school. Among the pupils of the ninth grade, its level is still very low and for that reason we did not come across a narrower correlation with other dimensions in our research.

As mentioned above, the first four dimensions of SR (CONSER, PROPOR, VARIABL, PROBAB) correlate together (with the majority of pairs correlating in a very strong way) and they therefore create the necessary background for the development of other, higher dimensions. In our research we have also found out that three of the dimensions (PROPOR, VARIABL a PROBAB) at the same time strongly correlate with UK (with UK also correlating slightly with the lowest dimension CONSER). On the basis of these findings we assume that the mastering of UK among students at the end of primary school is tightly linked with the development of more general skills at the level of the first four dimensions. We therefore think that dimensions PROPOR, VARIABL, PROBAB and component UK are important for the further development of learners in the STEM area.

Similarly, also foreign research points to the necessity to develop not only content understanding, but also scientific reasoning (Bao et. al., 2009). Positive correlations between student scientific reasoning abilities and measures of students' gains in learning science content have been reported also by Coletta and Phillips (2005). These findings support the consensus of the science education community on the need for students to develop an adequate level of scientific reasoning skills together with a solid foundation of content knowledge.

With regard to the weekly correlating pairs, our research suggests the link between RC and VARIABL. This fact can be explained by the dimension's being represented in the test by the biggest number of tasks (altogether 6) that belong to the most demanding ones in terms of reader's comprehension (the texts of these tasks were relatively extensive and demanded understanding of more complex texts).

A negative correlation appeared between RC and HYPDED. The tasks from the area of HYPDED are the most difficult ones and in addition to that quite demanding in terms of RC. The results of our research suggest that the better the reading skills pupils have, the lower level they achieve in the area of HYPDED. This may point to the fact that children have a quite well developed reading comprehension at the monitored age, but in the area of HYPDED the level is quite low.

No stronger relationship has appeared between C from CPS and SR dimensions, which is a considerably surprising finding rather than what we would expect here.

We can observe all the above-mentioned relations between the SR dimensions and the CPS components well-arranged in Figure 2. The arrows suggest relations between observed dimensions and components that correlate at the 5% level of significance.

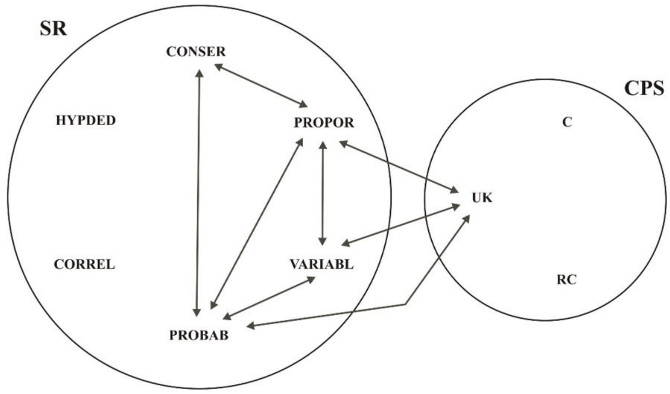

Figure 2: The scheme showing relations between SR dimensions and CPS components (source: own drawing)

Finally we explore how do individual components of CPS and individual dimensions of SR correlate with pupils' school performance in mathematics and physics. These subjects were selected for the study as they are the subjects in which abilities included in CPS and SR are significantly developed. On the basis previously conducted research (Shayer and Adey, 1993) we expected that some correlations would show.

In the Czech Republic, lower secondary school pupils are evaluated by grades 1 to 5 on a school report. Grade 1 corresponds to the best performance, 5 describes insufficient performance, failure. In Table 4 we present relative frequency of grades 1 to 5 in mathematics and physics in the studied sample of pupils.

Grade	Math	Physics
	Rel. freq. (%)	Rel. freq. (%)
1	13.04	8.70
2	73.92	69.57
3	13.04	21.74
4	0	0
5	0	0

Table 4: Relative frequency of grades 1 to 5 in mathematics and physics (source: own calculation)

Table 4 shows that pupils' school grades in physics are worse than in mathematics. However, there is no pupil having grade 4 or grade 5.

The correlations between the components of CPS, the dimensions of SR and pupils' school performance in mathematics and physics are presented in Table 5.

Mostly negative correlation coefficients in Table 5, that are, however, in most cases below the level of significance, suggest the indirect nature of the investigated dependences, that is the better the pupil performs in CPS, respectively in SR, the lower (i.e. better) grade he gains. Only grades in physics significantly correlate with the component C of CPS and the dimension CONSER of SR (bold types in the table 5). These results were quite surprising for us. That is why we investigated the correlation between the grades from mathematics and physics, where we would expect a stronger dependence quite rightly, as these two subjects are closely related to their focus and content.

However, even in this case only a very weak, statistically insignificant, correlation was shown. We believe that this fact is related to the size of the research sample and it should therefore be the subject of further research with a substantially a larger set of pupils.

Pair of Variables	*N*	Spearman *R*	R^2	*p*-level
MATH & RC	21	0.1092	0.0119	0.6376
MATH & UK	20	– 0.0032	0.0000	0.9893
MATH & C	20	0.0298	0.0009	0.9007
PHYS & RC	21	– 0.3791	0.1437	0.09013
PHYS & UK	20	– 0.0392	0.0015	0.8696
PHYS & C	**20**	**– 0.5243**	**0.2749**	**0.0176**
MATH & CONSER	18	– 0.2249	0.0506	0.3695
MATH & PROPOR	18	– 0.4020	0.1616	0.0982
MATH & VARIABL	18	– 0.1443	0.0208	0.5677
MATH & PROBABL	18	– 0.1956	0.0383	0.4366
MATH & CORREL	18	0.0000	0.0000	1.0000
MATH & HYPDED	18	– 0.0712	0.0051	0.7789
PHYS & CONSER	**18**	**– 0.6035**	**0.3642**	**0.0080**
PHYS & PROPOR	18	– 0.2495	0.0622	0.3181
PHYS & VARIABL	18	– 0.4095	0.1677	0.0915
PHYS & PROBABL	18	– 0.3159	0.0998	0.2016
PHYS & CORREL	18	– 0.4140	0.1714	0.0876
PHYS & HYPDED	18	0.0233	0.0005	0.9269

Table 5: Spearman correlation coefficients for the components of CPS, the SR dimensions and pupils' school performances (source: own calculation)

Conclusion

The results given in the previous chapter prove the legitimacy of the idea of exploring mutual relations between individual components of CPS and SR indicators and indicated the limits of our study, especially with respect to the investigation of dependence between individual CPS and SR indicators and pupil´s school performance in mathematics and physics. In the sufficiently wide research ($N = 200$) we will try to verify whether one of the components of CPS, the ability to use the existing knowledge, really correlates so strongly to the three dimensions of SR – proportional reasoning, control of variables and probability reasoning, as it has been shown by the pilot study described in this paper. The aim of the subsequent research will be to verify the influence intelligence as the fourth component of CPS on individual dimensions of SR and in addition to that to confirm or disconfirm a relatively surprising result of the pilot research that creativity does not correlate with SR dimensions. Among the subjects that we will take as a basis for the assessment of pupil' school performance we will include another key subject, namely mother tongue (Czech). On a sufficient larger research sample we want to verify again the dependence or the independence of individual components of CPS and individual dimensions of SR with pupil´s school performance in mathematics and physics. By means of demonstrating relations between both the constructs a path might be opened for teaching practice. It might show how to enable teachers to remove obstacles in pupils' problem solving more effectively, in particular with respect to courses in mathematics and physics.

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