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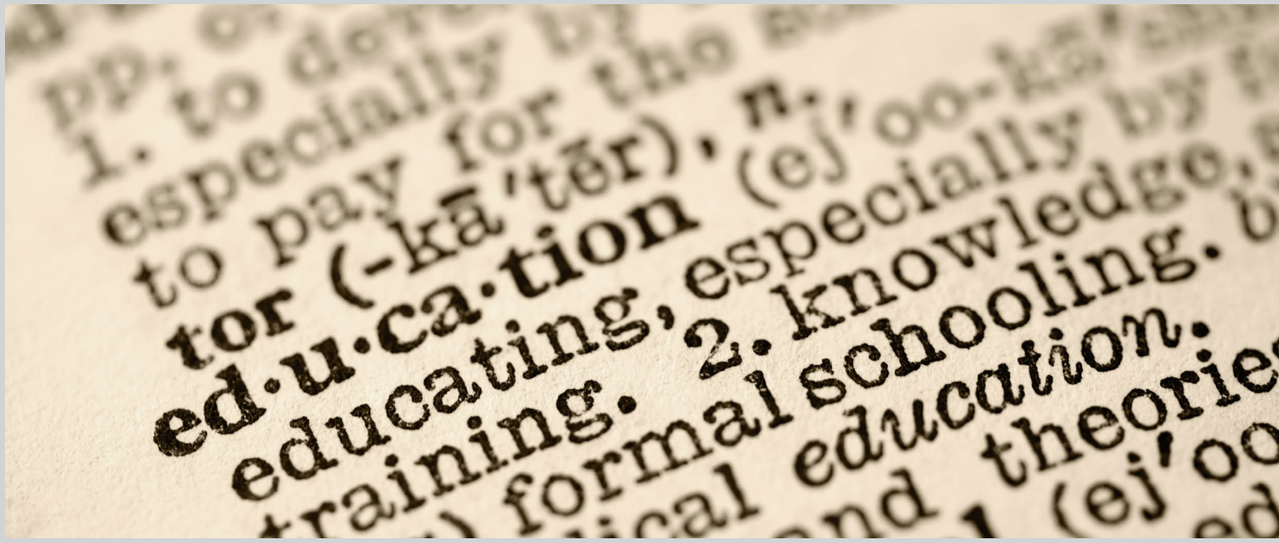

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Papers are published in English. A paper may comprise an empirical study using an acceptable research strategy, such as survey, case study, experiment, archival analysis, etc. It may contain a theoretical study aimed at advancing current theory or adapting theory to local conditions or it may arise from theoretical studies aimed at reviewing and/or synthesizing existing theory. Concepts and underlying principles should be emphasized, with enough background information to orient any reader who is not a specialist in the particular subject area.

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At the beginning of this new editorial, let me first look back to the 16th International Conference on Efficiency and Responsibility in Education (ERIE 2019) that took place in Prague on June 6 - 7, 2019. We had received articles from a diverse variety of higher education institutions from different countries, including China, Czech Republic, Hungary, Kazakhstan, Mexico, Romania, Russia, Slovakia, Turkey, Ukraine and the United States. As our commitment is to maintain the high quality of the accepted articles, the rejection rate reached 40.27%. We are glad that the conference editors, co-editors and reviewers keep the high standard of the review process and only articles that fulfil the conference high standards have been accepted for the conference proceedings. We are also glad that an interesting variety of topics were presented during the conference, such as research results related to quality evaluation of personnel and academic institutions, the link between education and labour markets, social responsibility in higher education, academic burnout, and many more. Last but not least, the participants had an opportunity to hear a keynote speech from Dr Bugay Turhan with the topic "Higher education quality in an era of competencies" and from Doc. Ing. Ludmila Dömeová, CSc. with the overview about "Generation Z at workplace". We are already thinking about the next year of the conference in which we would like to introduce several changes to improve its quality and services in order to expand into more countries and attract more participants.

In the newest issue of the ERIES Journal (Vol. 12, No. 2) we present three articles that all relate to one common topic: quality of teaching and innovative teaching approaches. As it is commonly known, students value courses that capture their interests and prepares them well for their future career. In most of the cases, research states that student evaluations of teaching are biased by the professor's personal characteristics. However, teachers must leave this common assumption and innovate their teaching practices no matter whether it is in mathematics, language or economics courses. Sometimes, it is necessary to break current routines to get outside the comfort zones. It may take time and extra effort, but it can be beneficial for the learning outcome of the students.

The first article "Evaluation of the Quality of Teaching from the Perspective of University Students" from Martina Fejfarová and Adéla Fajčíková from Czech University of Life Sciences Prague analyses variables that, from the perspective of university students, affect the quality of taught courses. The authors used a questionnaire survey including 450 respondents (24.2% males and 75.8% females) of the last years of master study programs in Economics and Management and Business Administration. Students were asked about characteristics that influence their perception of 11 taught courses in these study programs. The results showed that students' perception of a course is mainly impacted by whether they consider it beneficial, interesting and whether it is easy to understand. With respect to the teacher's skills, the most important is the ability to explain and train the course content. These findings can be beneficial to formulate recommendations for teachers, for both new coming and current workforce, as well as for educational decision-makers.

The second article "Lesson Study for Professional Development of English Language Teachers: Key Takeaways from International Practices" from *Özgehan* Uştuk and *İrem Çomoğlu* from Dokuz Eylul University provides a review study of professional development practices for foreign language teachers. According to the authors, the main problem lies in missing connection between education policymakers and the teachers. This may cause several problems as if language teachers are not involved as active decision-makers regarding their own professional development practices, the outcomes may not be as desirable as policymakers plan. The authors conclude that lesson study has a potential to help EFL teacher to integrate language and content in a balanced way provided that administrative support is ensured. Moreover, lesson study practice can improve teachers' collaborative working culture and pedagogical literacy. Last but not least, lesson study practice can be used to transform in-service teacher education both in terms of content and form. Improving lesson study practices can lead to effective and sustainable language teachers' professional development.

The third article "The Strategy the Use of False Assumption and Word Problem Solving", the authors Petr Eisenmann, Jiří Přibyl and Jarmila Novotná from University of Jan Evangelista in Ústí nad Labem and Charles University deals with the Use of the false assumption problem solving strategy. The authors demonstrate that it is helpful to reiterate the process of development of the concept of a variable to provide to help pupils to eliminate usual difficulties when solving word problems using linear equations. The analysis is based on a sample formed by 147 pupils eight grade classes at lower secondary schools in three different cities in the Czech Republic. The results of the experiment confirmed that the introduction of the Use of false assumption strategy helps pupils to construct equations more successfully when solving word problems and, thus, this strategy facilitates the whole solving process.

We would like to thank all reviewers who contributed to this second issue of 2019, as well as we would also like to thank all authors who have submitted their manuscripts to ERIES Journal. We hope that all our readers will find this issue interesting, and we also hope that ERIES Journal will contribute with new insights, research methods and analyses to the field of efficiency and responsibility in education as it has contributed so far.

Sincerely

Martin Flégl
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CONTENT

Full research paper

EVALUATION OF THE QUALITY OF TEACHING FROM THE PERSPECTIVE OF UNIVERSITY STUDENTS

Adéla Fajčíková[1]
Martina Fejfarová[1]✉

[1] Faculty of Economics and Management, Czech University of Life Sciences Prague, Czech Republic

✉ fejfarovam@pef.czu.cz

ABSTRACT

The concept of quality in tertiary education is important not only for education providers but also for society as a whole which expects the education to have a high level of quality and wants to maintain its trust in the education system on the national and international level. The objective of this paper is therefore to identify variables that, from the perspective of university students, affect the quality of taught courses, and use the results to formulate recommendations that would improve the quality of teaching at universities. Primary data was obtained through a quantitative survey among students in the Master's programme at a selected public university involving 450 respondents. The results showed that the overall rating of a course is more strongly influenced by variables related to the course's outcome, content and concept than the rating of the teacher. The main value lies in the general overview of the variables influencing the perception of the quality of taught courses by students that is beneficial for teachers who are preparing a course and also for higher education institutions developing a methodology for evaluating the quality of teaching and teachers.

KEYWORDS

Course, quality, quantitative survey, tertiary education, university students

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Highlights

- *The evaluation of the quality of teaching from the perspective of university students via a quantitative approach.*
- *The general overview of the variables influencing the perception of the quality of taught courses by students.*
- *Students give positive ratings even to courses that are very difficult and require more self-study if they are considered beneficial.*
- *The overall rating of a course is more strongly influenced by variables related to the course's concept, content and outcome than the evaluation of the teacher.*

INTRODUCTION

Higher education institutions as the highest level of the education system are leading centres of education, independent learning and creative activity, and play a crucial role in the scientific, cultural, social and economic development of society (Lesjak, 2018; Scott, Gallacher and Parry, 2017).

After years of preparations and negotiations, on 2 May 2016 a new amendment of the Act on Higher Education Institutions and on Amendments and Supplements to Some Other Acts (1998) became effective in the Czech Republic which, among other things, addresses internal evaluations of the quality of teaching, creative, and related activities of a higher education institution, consisting of a) the application of standards and methods of internal evaluation of the quality of teaching, creative, and related activities of a higher education institution; b) elaborating a report on internal evaluations of the higher education institution based on educational, creative, and related activities of the higher education institution describing attained qualitative outputs of the higher education institution together with measures accepted for removing potential deficiencies (to be published at intervals defined by the institution's internal regulations, but no less frequently than once in 5 years); and c) making the report and additions available for the bodies and members of bodies of a higher education institution and its parts as well as to the Accreditation Bureau and the Ministry of Education (Higher Education Institutions and on Amendments and Supplements to Some Other Acts, 1998). At the same time, the Government Regulation on Standards for Accreditation

in Higher Education was published; this regulation became effective on 1 September 2016 and determines the standards for institutional accreditation, standards for accreditation of a degree programme, standards for accreditation of the habilitation procedure and standards for accreditation of the procedure for appointment of a professor. The standards for institutional accreditation among other requirements also include the existence of an internal quality assurance and assessment system of educational, creative, and activities related to the higher education institution. In educational, creative, and related activities, the higher education institution must have certain indicators set up that allow it to monitor a success rate at the admission procedure, drop-out rate in the degree programme, rate of completion of studies within a degree programme and rate of graduate employment (Government Regulation no. 274/2016 Coll.).

The fundamental roles of higher education institutions include education, scientific research and the 'third role' which covers a broad range of activities not included in either of the previous two traditional categories. Recent literature has also started talking about a 'fourth role' which according to Pawłowski (2009) contributes to the development of regions where the institutions are located. Unlike some economists who strictly insist on applying a cost-benefit analysis to tertiary education spending, Bowen (2018) claims that the non-monetary benefits of a tertiary education far outweigh the monetary component. According to a strategic document of the Ministry of Education, Youth and Sports (2015) titled Higher Education Development Framework for 2020, one of the greatest challenges of the upcoming period is a sharp decline in the number of students from the traditional population caused by demographic development. For this reason, it is necessary to constantly seek answers to the question how to efficiently secure and keep increasing the quality of all the activities of higher education institutions and how to increase the value and relevance of education for each student. According to Chui et al. (2016) and Ming (2010), the market of higher education institutions is highly competitive, unstable and turbulent. If a higher education institution is to weather all economic and social change, it needs to understand the needs of its students better to be able to attract and retain them (Wiese, Van Heerden and Jordaan, 2010). The issue of improving the quality of education is, therefore, a hotly discussed topic on all levels of society, as the quality of education has a major impact on students, teachers, the government and public as a whole (Leeuwenkamp, Brinke and Kester, 2017).

The issues of maintaining the quality of the education system are currently in the centre of attention and the subject of many specialised papers. The concept of quality in tertiary education is important not only for the institutions that provide education but also for society as a whole which expects the education to have a high level of quality and wants to maintain its trust in the education system on the national and international level (Prisacariu, 2015).

The quality of a higher education institution can be assessed either from the perspective of the quality of the teaching itself or by evaluating the results and knowledge of the students (Pereira, Araujo and Machado-Taylor, 2018), examining their progress in learning and skill development by comparing the situation before and after the educational activities (Ďurišová, Kucharčíková and Tokarčíková, 2015). This paper focuses on quality evaluation from the perspective of students whose expectations and needs are what each higher education institution should primarily strive to fulfil.

The quality of education is influenced by many factors. Devadoss and Foltz (1996) and Dolton, Marcenaro and Navarro (2003) analysed the relationship between student attendance and academic performance. The results of the research show that attendance at lectures, seminars etc. clearly brings numerous benefits to students (Stanca, 2006). Lindstadt (2005) and McCluskey, Bynum and Patchin (2004) agree that the critical factors that decide on whether students take part in the learning process or not include among individual, family or social aspects also factors related to the tertiary education institution as such – its structure, rules, environment or employees. Because teachers are the main bearers of knowledge in a higher education institution, they are also among the main factors influencing the institution's quality (El-Hilali, Al-Jaber and Hussein, 2015). For this reason, it is important to regularly evaluate the efficiency of teachers and identify their strengths and weaknesses which may help improve their personal performance but also play an important role for the institution itself when drafting its policy for hiring teachers and conducting their professional development (Medallon and Martinez, 2014). Adnot et al. (2017) and Stronge (2018) claim that teachers have a strong and lasting impact on students, influencing not just what the students learn but also how and how often, what is their attitude towards their studies and the institution and how they influence and are influenced by their environment. According to Medallon and Martinez (2014), teachers are typically evaluated in four dimensions: personal characteristics, professional competence, classroom management and the teacher-student relationship. Stronge (2018) emphasises beside the teacher's professional knowledge and skills also the ability to plan the teaching, to teach the course content, objectively assess, create a positive atmosphere and act professionally.

Hoang et al. (2016) claim that the existence and sustainable growth of any organisation depends on customer satisfaction. It can be said, therefore, that the objective of any higher education institution that wishes to succeed in the competitive environment is to provide a high quality of services, improve student engagement and seek ways how to fulfil the needs and wishes of its students better than other institutions (Orindaru, 2015).

The objective of this paper is therefore to identify variables that, from the perspective of university students, affect the quality of taught courses, and use the results to formulate recommendations that would improve the quality of teaching at universities.

We formulated the following research questions:

- Is there a significant dependency relationship the overall rating of a course and the variables such as age, gender, study programme or other identification variables?
- Is the rating of a course significantly influenced by regular attendance of the student at the lectures?

- What variables have the most significant impact on the perceived quality of teaching from the perspective of university students?
- Is the overall rating of the course primarily influenced by variables relating to the course itself, the method of teaching the course or the personality of the teacher?

The paper is structured as follows: First, the summary of the current knowledge is presented in Introduction. The section Materials and Methods describes the used research methods and statistical techniques in this paper. The gathered findings are assessed in Results. The achieved and presented results are then elaborated and compared with those of international studies in Discussion. This part also contains subsequent recommendations for teachers who are preparing the educational courses, teachers who are interested in improving the students' interest in existing courses and for higher education institutions and identifies the benefits and limitations of the paper. The section Conclusion summarises the main findings.

MATERIALS AND METHODS

Primary data was obtained in a quantitative survey involving 450 respondents. The participants were purposefully selected: the questionnaire respondents were the students of the two largest Master's programmes at the Faculty of Economics and Management, Czech University of Life Sciences Prague (FEM CULS Prague): Economics and Management (EM) and Business Administration (BA). The questionnaire only targeted students in the fourth and fifth year of the full-time study programme in Czech. The research was carried out in the academic years 2016/2017 and 2017/2018; the students were asked about characteristics that influence their perception of 11 taught courses. In the survey, we asked 9 questions consisting of 6 identification questions and 3 summary research questions which comprised individual claims concerning the assessment of the course, teaching and teacher. In the identification questions, the respondents could choose one answer; in research questions, they indicated their agreement with individual claims using the five-grade Likert scale (strongly agree, agree, neutral, disagree, strongly disagree). From the total number of 450 respondents, 180 (40%) are already working in a field of education where they can apply knowledge from the taught courses. 237 (52.7%) of respondents are planning to work in such fields; 90 (20%) respondents do not know where they want to work and 123 (27.3%) respondents are planning to work in a completely unrelated field of education. The structure of respondents is shown in Table 1.

Gender	Men		Women		Total
	109 (24.2%)		341 (75.8%)		450 (100%)
Age category	**21-23 years**	**24-26 years**	**Over 26 years**		**Total**
	137 (30.4%)	227 (50.4%)	86 (19.1%)		450 (100%)
Study programme	**BA**		**EM**		**Total**
	290 (64.4%)		160 (35.6%)		450 (100%)
Lecture attendance*	**25%**	**50%**	**75%**	**100%**	**Total**
	144 (32%)	58 (12.9%)	97 (21.6%)	151 (33.6%)	450 (100%)

*Note: *The decision on whether attendance at lectures would be mandatory or not is made by the course guarantor. Because seminars are always mandatory, attendance at seminars was not examined.*

Table 1: Structure of respondents, 2016-2018 (source: own survey)

After clarification of the key dependent and independent variables, we formulated 39 partial null hypotheses assume no relationship between the overall rating of a course and basic identification variables ($H_0$1–$H_0$5), partial variables influencing the overall rating of the course summarily called 'course evaluation' ($H_0$6–$H_0$15), partial variables influencing positive ratings of the teaching summarily called 'teaching evaluation' ($H_0$16–$H_0$26) and partial variables influencing positive ratings of the teacher summarily called 'teacher evaluation' ($H_0$27–$H_0$39). The variables were ranked by importance depending on the examined strength of the relationship.

The statistical software used to evaluate the data and calculate independence tests (Pearson's chi-square test of independence) was IBM SPSS Statistics 24. When the obtained *p*-value was below the significance threshold $\alpha = 0.05$, the null hypothesis was rejected. The strength of the relationship was examined using the Cramer's *V* coefficient using the scale given by De Vaus (2014) as follows: 0.10–0.29 (low to moderate), 0.30-0.49 (moderate to substantial) and 0.50–0.69 (substantial to very strong).

RESULTS

Firstly, we tested the impact of basic identification variables on the overall rating of a course, then we paid attention to variables connected with course evaluation, teaching evaluation and teacher evaluation that also might influence on the overall rating of a course.

Impact of basic identification variables on the overall rating of a course

Pearson's chi-square test of independence showed that there is a relationship between the overall rating of a course, i.e. whether the course meets students' expectations, and their age ($H_0$3: $p = 0.000$, $V = 0.185$), their employment in the field of

education ($H_0$4: p = 0.001, V = 0.208) and regular participation at lectures ($H_0$5: p = 0.000, V = 0.233). It was also examined that the variables of the study programme ($H_0$1) and gender ($H_0$2) have no impact on the overall rating. Based on the above, $H_0$1 and $H_0$2 were not rejected on the level of significance threshold α = 0.05. Hypotheses $H_0$3–$H_0$5 were rejected because there is a relationship between the overall rating of a course and age, employment in the field of education and attendance at lectures. The results are shown in Table 2.

No.	Variable	*p*-value	Cramer's *V*	Strength of relationship
$H_0$1	Study programme	0.961	-	-
$H_0$2	Gender	0.481	-	-
$H_0$3	Age	0.000	0.185	Low
$H_0$4	Employment in the field of education	0.001	0.208	Low
$H_0$5	Attendance at lectures	0.000	0.233	Low

Table 2: Basic identification variables, 2016-2018 (source: own survey)

Generally speaking, with increasing age of students tend to give their courses better ratings and feel that their expectations are being fulfilled. Similarly, courses are rated better by students working in a field of education where they can apply the knowledge from the course and who regularly attend lectures. While only 36.1% of students whose attendance at lectures was below 25% gave the corresponding course a positive rating, students who attended regularly gave positive answers in 74.8% of cases. The results show that students who attend lectures regularly give the course a better rating.

Impact of partial variables summarily called 'course evaluation' on the overall rating of the course

We also examined a relationship between 10 partial variables summarily called 'course evaluation' and the overall rating of the course (i.e. whether the course meets students' expectations).

The results are shown in Table 3.

No.	Variable	*p*-value	Cramer's *V*	Strength of relationship
$H_0$6	Course is beneficial	0.000	0.468	Substantial
$H_0$7	Course is interesting	0.000	0.439	Substantial
$H_0$8	Course is important for practice	0.000	0.346	Moderate
$H_0$9	Course is difficult	0.428	-	-
$H_0$10	Course is provided within study materials	0.000	0.250	Low
$H_0$11	Study materials are adequate	0.000	0.285	Moderate
$H_0$12	Course is well integrated with the study programme	0.000	0.352	Moderate
$H_0$13	Credit requirements are adequate	0.000	0.285	Moderate
$H_0$14	Examination requirements are adequate	0.000	0.254	Low
$H_0$15	Adequate time allocation	0.076	-	-

Table 3: Partial variables summarily called 'course evaluation', 2016-2018 (source: own survey)

Based on the above, $H_0$9 and $H_0$15 were not rejected on the level of significance threshold α = 0.05. Hypotheses $H_0$6–$H_0$8 and $H_0$10–$H_0$14 were rejected because there is a relationship between the overall rating of a course and the partial variables summarily called 'course evaluation'. Based on the examined strengths of the relationships, it can be summarised that students generally give a course a better rating if they consider it beneficial, interesting, well integrated within the study programme and important for practice (V = 0.468–0.346). Statistically less important were variables that relate to the successful completion of the course, i.e. scope and quality of supporting materials or adequacy of the requirements for the credit/exam. The results also showed an interesting finding – the lack of any relationship between a positive rating of a course and its difficulty level or adequate time allocation. From the above, it can be concluded that students prefer courses that develop their knowledge and skills and prepare them for their future job rather than courses that are easy to pass. For this reason, students give positive ratings also to courses that are very difficult or require more additional self-study.

Impact of partial variables summarily called 'teaching evaluation' on the overall rating of a course

Based on the examined relationship between the overall positive rating of a course and agreement (strongly agree, agree,

neutral, disagree, strongly disagree) with verbal claims concerning teaching evaluation provided below, the variables influencing course rating were ranked by importance. These were 11 partial variables summarily called 'teaching evaluation'. The course meets students' expectations particularly when they are satisfied with the content of seminars and how they are conducted if they consider the lectures easy to understand and logically structured and find the method of teaching appropriate. These variables can be considered substantial because the strength of the relationship as expressed by Cramer's *V* coefficient was between 0.456 and 0.419. Students also give more positive ratings to courses in which they are satisfied with how the lectures are conducted if the teacher is helpful and stimulates learning if the seminars complement lectures well, there are a clearly defined objective and course requirements or if they are satisfied with the pace of the teaching ($V = 0.399–0.323$). A particularly surprising result was obtained for the variable 'external experts are involved'. Even though the students give very positive ratings to courses that are applicable in practice, there was only a weak relationship between the overall rating of a course and the involvement of external experts ($V = 0.220$). The results are shown in Table 4.

No.	Variable	*p*-value	Cramer's *V*	Strength of relationship
$H_0$16	Course objective and requirements are clearly defined	0.000	0.378	Moderate
$H_0$17	Lectures are well conducted	0.000	0.399	Moderate
$H_0$18	Lectures are easy to understand	0.000	0.442	Substantial
$H_0$19	Lectures are logically structured	0.000	0.436	Substantial
$H_0$20	Pace of the teaching is satisfactory	0.000	0.323	Moderate
$H_0$21	Appropriate method of teaching the lectures	0.000	0.419	Substantial
$H_0$22	External experts are involved	0.000	0.220	Low
$H_0$23	Seminars adequately complement the lectures	0.000	0.386	Moderate
$H_0$24	Seminar content is satisfactory	0.000	0.456	Substantial
$H_0$25	Appropriate method of teaching the seminars	0.000	0.443	Substantial
$H_0$26	Teacher is helpful and stimulates learning	0.000	0.396	Moderate

Table 4: Partial variables summarily called 'teaching evaluation', 2016-2018 (source: own survey)

Based on the above, the hypotheses $H_0$16–$H_0$26 were rejected because there is a relationship between the overall rating of a course and the partial variables summarily called 'teaching evaluation'.

Impact of partial variables summarily called 'teacher evaluation' on the overall rating of a course

As the last step, we examined the relationship between 13 partial variables summarily called 'teacher evaluation' and the overall rating of a course. Based on the obtained results, we can summarise that the overall positive perception of a taught course more strongly depends on the evaluation of the teaching (see Table 4) than the rating of the teacher, even though there were still some relationships. When assessing teachers, the factors that students consider the most important are the ability to present the content and practice it and the ability to capture students' attention and stimulate learning ($V = 0.423–0.394$). It's also important for students to have teachers who are able to connect the taught content to practice ($V = 0.354$) and evaluate students objectively ($V = 0.331$) than whether they act in a professional manner ($V = 0.286$) and whether their behaviour towards students is proper ($V = 0.264$). The detailed results are shown in Table 5.

Based on the above, the hypotheses $H_0$27–$H_0$39 were rejected because there is a relationship between the overall rating of a course and the partial variables summarily called 'teacher evaluation'.

DISCUSSION

Because the community of a higher education institution consists of both students and teachers, the interaction between these two groups seems to be an essential factor for the modification of education as a step for its development (Nada and Hamed, 2018). Research by Adnot et al. (2017) proved that when low-performing teachers are replaced by more efficient colleagues, the positive impact on study performance of students may be very significant. Every higher education institution whose main objective is to provide services of high quality should, therefore, be interested in the efficiency of its teaching staff, differentiate between exceptional and average teachers and systematically examine how they are perceived by the students (who are the higher education institutions' customers).

Our research examined that students place the greatest emphasis on the benefits of the taught course and give positive ratings even to courses that are very difficult and require more work in preparation. This is confirmed by Rafaila and Duta (2015) who claim that students are willing to self-study and devote maximum effort to learning when the course and taught content contribute to their professional and personal development.

Based on the obtained data, we can formulate the following recommendations for teachers who are preparing the educational courses, teachers who are interested in improving the students' interest in existing courses and for higher

No.	Variable	*p*-value	Cramer'*V*	Strength of relationship
$H_0$27	Teacher acts in a professional manner	0.000	0.286	Moderate
$H_0$28	Teacher uses modern teaching techniques	0.000	0.293	Moderate
$H_0$29	Teacher uses modern technologies	0.000	0.286	Moderate
$H_0$30	Teacher stimulates learning	0.000	0.394	Moderate
$H_0$31	Teacher has the ability to capture student's attention	0.000	0.397	Moderate
$H_0$32	Teacher visualizes the presented information	0.000	0.340	Moderate
$H_0$33	Teacher creates a positive and friendly atmosphere	0.000	0.309	Moderate
$H_0$34	Teacher behaves appropriately	0.000	0.264	Low
$H_0$35	Teacher has the ability to present the content	0.000	0.423	Substantial
$H_0$36	Teacher has the ability to connect the taught content to practice	0.000	0.354	Moderate
$H_0$37	Teacher adequately practises the subject matter	0.000	0.404	Substantial
$H_0$38	Teacher is dedicated to students	0.000	0.309	Moderate
$H_0$39	Teacher evaluates objectively	0.000	0.331	Moderate

Table 5: Partial variables summarily called 'teacher evaluation', 2016-2018 (source: own survey)

education institutions aiming to evaluate the quality of teaching and develop a methodology for teaching quality:

- Regularly evaluate the quality of teaching and the efficiency of teachers. Keane (2015) says that institutions typically carry out quantitative surveys among students who are able to rate their own experience with the teaching. He however also points out that these surveys have issues with validity and readability and recommends applying for peer review.
- When preparing a course, pay particular attention to the logical structure of the lectures and seminars to ensure that knowledge and experience are provided in a clear and easy to understand manner.
- Explain the subject matter carefully and patiently, verify that it was understood and assign exercises focused on past content. The research results of Alauddin and Kifle (2014) coincide with our findings that good course organisation, clear presentation and explanation are among the most important factors influencing students' satisfaction and learning outcomes.
- Combine theoretical learnings with examples from practice or alternatively let students gain and deepen their knowledge by learning from their own experience. The importance of focusing on the students' personal experience and their ability to apply their knowledge in practice also underline Arsenijević and Maljković (2016).

The main benefits of this paper lie in its general overview of the variables influencing the perception of the quality of taught courses by students. Course evaluation included many variables such as the requirements of the course, benefits for practice, the structure of the lectures and how they are conducted, style and method of teaching, the range of supporting materials and the lecturer's characteristics. This paper may, therefore, be useful for teachers who are preparing a course or for higher education institutions developing a methodology for evaluating the quality of teaching and teachers and for creating and innovating degree programmes for accreditation and re-accreditation needs. The main limitation of the paper is that the quantitative survey was carried out at only one public university. Even though the number of questionnaire respondents was fairly large (n = 450), they were all students in the Master's study programme. Subsequent research should, therefore, compare the perspective of students of private and public universities as well as of students in Bachelor's and Master's programmes. Such a comparison could reveal different expectations of students from private and public universities, as well as draw attention to the educational needs that may develop during their studies.

CONCLUSION

The quality of university education has a major impact on students, teachers and society as a whole, which is why the topic of teaching quality assessment may be considered highly relevant today and crucial for the future competitiveness of every country. Our research showed that the overall rating of a course is more strongly influenced by variables related to the course's concept, content and outcome than the evaluation of the teacher. The perception of a course by students is the most strongly impacted by whether they consider it beneficial (0.468) and interesting (0.439), whether it's easy to understand (0.442) and logically structured (0.436) and whether the form of presentation and the content of exercises are appropriate (0.456). In teacher evaluation, the most important skill in our analysis was the ability to explain (0.423) and train (0.404) the course content. It was also examined that students give positive ratings even to courses that are very difficult and require more self-study if they are considered beneficial. In our respondent sample, we found no relationship between course rating and gender or studied programme. On the other hand, there was a weak relationship between the respondents' age, their attendance at lectures of the course and whether they work in a field related to the content of the course.

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LESSON STUDY FOR PROFESSIONAL DEVELOPMENT OF ENGLISH LANGUAGE TEACHERS: KEY TAKEAWAYS FROM INTERNATIONAL PRACTICES

Özgehan Uştuk[1]✉
İrem Çomoğlu[1]

[1] Dokuz Eylul University, Institute of Educational Sciences, Turkey

✉ oustuk@balikesir.edu.tr

ABSTRACT

Even though various professional development practices are provided for language teachers worldwide, these practices are still considered ineffective in addressing teachers' needs in their classrooms. The effectiveness of these practices is stalled when teachers do not actively engage in their professional development processes. When language teachers are not involved as active decision makers regarding their own professional development practices, the outcomes may not be as desirable as policymakers plan. Considering the deficiencies of the current language teacher professional development practices in Turkey, this paper provides a systematic review of lesson study as a professional development model for language teachers which promotes agency and reflective practice. Comparing the current practice of lesson study in Turkey to the international practices reported in the literature, this review suggests that lesson study has potential to support teacher professional development in the country in alignment with the educational visions of decision makers. Given the empowering dimension of lesson study model both in terms of content and form, it can be adopted as a model for effective and sustainable language teacher professional development.

KEYWORDS

Efficiency, English as a foreign language, lesson study, professional development, teacher

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Highlights

- *Lesson study has potential to support teacher professional development in alignment with the educational visions of decision makers.*
- *Lesson study practice can improve teachers' collaborative working culture and pedagogical literacy.*
- *Lesson study practice can be used to transform in-service teacher education both in terms of content and form.*

INTRODUCTION

Constantly evolving demands and innovations in language learning and teaching have made career-long professional development of teachers an essential element for a sustainable language education policy. To support the professional development of language teachers, many training modules are provided in local, regional, national, and international contexts. However, the paradigmatic alignment of these attempts in terms of actual needs, form, and content is a matter of concern in this volatile world.

Khong and Saito (2014) put forward strong criticism with regard to conventional professional development practices. They problematized that such practices do not respond to the actual requisites of teaching and learning ecologies. Besides, they are one-off events and limited to a couple of days, which results in a minor or no influence on language teachers' beliefs and teaching philosophies.

Admittedly, the problems with regard to professional development for foreign language teachers, who teach English as an additional language to students' native language(s) in

Turkey are the same as the global ones. Atay (2007) criticized professional development practices to be one-shot events with a top-down understanding rather than bottom-up. In an exploratory study, Korkmazgil and Seferoglu (2013) found that English as a foreign language (EFL) teachers' professional development practices are mostly in the form of in-service seminars and workshops about a particular topic decided by other decisionmakers rather than teachers. Furthermore, they asserted that the content of these events does not match EFL teachers' perceived needs. Similarly, Oruç-Ertürk, Gün, and Kaynardağ (2014) found that professional development practices for EFL teachers usually have not authentic and situated goals.

Within the scope of professional development practices in Turkey, Uysal (2012) evaluated a one-week in-service teacher training program for primary school language teachers offered by Ministry of National Education (MoNE) in Turkey to explore its effect on teachers' attitudes, knowledge-base and classroom practices. Findings indicated that although the teachers' attitudes towards the course are positive in general, the one-shot program disregards the contextual needs of teachers and has limitations especially in terms of its planning and evaluation phases. Similarly, Koç (2016) explored the views of 32 elementary English language teachers who have participated in various in-service teacher-training activities organized by MoNE. The majority of the participant teachers complained that the activities offered were not relevant to their needs. For effectiveness in teacher professional development, Koç proposed a model which incorporates the identification of teacher needs, distance learning mode, active learning, a collaboration between MoNE and Higher Education Council, and assessment of program effectiveness.

Findings in Koç's study (2016) are in alignment with Darling-Hammond, Hyler and Gardner's (2017: v-vi) assertions regarding the key features of effective teacher professional development. According to them, effective teacher professional development practice is content focused, incorporates active learning, supports collaboration, uses models of effective practice, provides coaching and expert support, offers feedback and reflection, and is of sustained duration.

Diagnostic studies mentioned above demonstrate that the current teacher professional development models in Turkey do not incorporate all of these elements. Given the pitfalls of the current teacher development practices in the country, MoNE has taken action to update and regulate the teacher professional development issue. In a recent report by MoNE (2018), the improvement of professional competencies of foreign language teachers is underlined as one of the key actions to take. However, the question of "how to fulfil these actions" is still unearthed and of critical importance.

In tandem with this, the current study provides a review on a specific teacher professional development model, *lesson study*, and recommends it as an alternative to be considered in Turkey. Li and Wang (2018: 123) stated that rigorous literature reviews and the conclusions reached in them can serve as a basis for practitioners and policy-makers to make decisions of how to meet the needs of different stakeholders. Thus, in this paper, the authors discuss the state-of-the-art practice of lesson study with worldwide instances and problematize how it can be used to maintain a sustainable professional development model for language teachers in Turkey. This article is based on the doctoral dissertation of the first author.

A brief introduction to lesson study

Even though lesson study is not an old form of teacher professional development in the western world, according to Dotger (2015) the earliest practices date back to the second half of the 19th century in Japan. As a way for Japanese educators to enable teachers to study their teaching practice and investigate the transformative nature of collaboratively developed curricula (Baba, 2007), lesson study was introduced to the western educational domain in the 1990s with prolific scholars such as Stigler and Hiebert (1999) and Lewis (2004).

A systematic analysis of lesson study by Lewis (2000) framed the main characteristics of this model, which helped us to understand why lesson study is distinctive in form. She suggested that lesson study is a process-based model to scaffold teachers while they collaboratively plan a research lesson that is taught by one participant-teacher while others observe it. Moreover, they are planned collaboratively and address an educational goal or vision determined by participant teachers. The process is concluded with a discussion of the teaching practice (Lewis, 2000).

In more recent research, several models have been developed and used. One of them is Dudley's (2013) lesson study process, which starts with an initial meeting of the lesson study group. In this meeting, the group members decide what they want to improve regarding their teaching, and they plan the first research lesson jointly. When one of the group members teaches the research lesson in a target classroom, the other members observe pupils and may interview them after the lesson. Based on the data from interviews, observations, and teacher's reflections, the participants discuss the first research lesson and develop it to the second one. They follow a similar process until the group shares their findings regarding the whole process. In another practical model by Cajkler and Wood (2016a), a more recursive process is introduced. According to them, lesson study process starts by identifying a learning challenge in a specific educational context. Later the group members collaboratively plan a research lesson to address this challenge and one member teaches it while the others observe specifically focusing on student learning. After evaluating the research lesson by the help of feedback on student learning, the research lesson is revised for re-teaching. Cajkler and Wood (2016a) proposed that the process ends by revisiting the learning challenge, to transform it if needed, and to proceed to the following lesson study cycle.

These two models are based on the transformative understanding that promotes teachers with an agency who are responsible for their own learning and teaching. That is the opposite of traditional teacher professional development practices in which teachers are passive learners receiving the

content delivered by external experts. Research on lesson study shows that one of the most empowering influences of lesson study is to help the teachers develop agency in their teaching practice (Cajkler and Wood, 2016b; Lamb and Aldous, 2016). Another prominent result of the prior research is the promotion of reflective teaching practice with lesson study. This vein of research pointed out that lesson study model supports teachers' engagement in reflective practice (Olteanu, 2016; Ricks, 2011; Soto Gómez, Serván Núñez, and Caparros-Vida, 2016), which transforms teachers' understanding of student learning "because the issues they investigate are meaningful to their local teaching context" (Tasker, 2011: 221). Likewise, Johnson (2009) suggested that lesson study is an inquiry-based, teacher-directed, collaborative, and non-evaluative model that is grounded in actual teaching practices.

In the light of growing interest in transformative pedagogy and bridging research-pedagogy gap, studies reported lesson study experiences presented lesson study in terms of its efficiency. Sato and Loewen (2019) discussed communities of practice as one of the most feasible and efficient ways to establish transformative dialogue between researchers and teachers and teachers among themselves. Accordingly, Dudley (2014) reviewed that lesson study has potential to change the institutional norms and dynamics by creating communities of practice for the sake of safe and trustworthy environments for teachers to engage in research-driven knowledge and their peers' experiences. Similarly, Saito et al (2014) discussed lesson study as a part of sustainable school reform and proposed the term lesson study for learning community. According to the authors, lesson study's efficiency lies on its emphasis on long-term practice (as a cycle generally extends throughout weeks of implementation) and on effective professional learning of teachers; that is, professional development is continuing, social, and related to practice (Saito et al., 2014: 2).

On the other hand, research underlines several pitfalls for those who would like to implement lesson study. To illustrate, Gero (2015) argued that the implementers of lesson study should be well-aware of its bottom-up paradigm and avoid the temptation of a hierarchical version of it in which teaching challenges and goals are determined by administrators, mentors or policy-makers; whereas participant-teachers are "appointed" to follow the process. Furthermore, Bjuland and Mosvold (2015) highlighted the importance of an experienced lesson study facilitator to coach teachers throughout the process. They warned that inexperienced teachers or pre-service teachers may have difficulty in observing student learning, and they can be lured to theoretical discussions rather than the practical basis of lesson study. Other very important factors that may "contaminate" the optimal result of lesson study are the lack of external administrative support (Lo, 2009; Tasker, 2011) and failure in giving constructive feedback (Cajkler, et al., 2013; Parks, 2009).

Despite the highlighted pitfalls, lesson study is potentially a collaborative (Cajkler *et al*., 2013; Cajkler, et al., 2015), inquiry-based (Johnson, 2009) and bottom-up (Dudley, 2013) model. As a model that was proposed for sustainable and efficient school reform (Saito et al, 2014), the authors reviewed lesson study within the context of foreign language teaching practice.

The authors specifically focused on lesson study in action; in that, how it has functioned in particular contexts may lead educational leaders who may include teachers, department chairs, administrators, and policy-makers all around the world. Accordingly, this paper brings the discussion to one step further than the question of "is lesson study effective in school reform?"; instead, it aims to review studies based on practice and discuss the implications of these practices for school reform in Turkey. Arguably, focusing on a specific context embedded in the wider context of lesson study applications is not a limitation for the current paper's transferability. This study is part of an on-going project that discusses the efficiency of lesson study at a foreign language instruction context in Turkey. By doing so, the conversation may transcend its context and influence similar reformist movements. Therefore, it is not only a review paper, but a start of a longer conversation for a sustainable and efficient teacher professional development policy.

In tandem with this background, the authors argue that this model can be discussed within the context of a recent reformist movement in Turkey that specifically aims to empower foreign language teachers through various reforms in both pre-service and in-service education (MoNE, 2018). By systematically analyzing the existing lesson study practice in Turkey with the lens of international practices, this paper provided implications for emancipating a lesson study-oriented language teacher professional development policy.

MATERIALS AND METHODS

A reviewing protocol was developed and followed to enact a systematic review. First, the authors used Scopus and ERIC databases with keywords "*lesson study"* (in-between quotation marks), "*language"*, and "*education"*. Scopus database was chosen since it includes *International Journal for Lesson and Learning Studies*, a major journal supported by *World Association of Lesson Studies* (WALS). The authors also decided to utilize ERIC as it is an authoritative database of indexed education literature. 21 articles on Scopus database and 64 peer-reviewed articles on ERIC database were reached on January, 22, 2019. Second, the authors introduced a set of inclusion criteria to find the relevant international research among 85 articles reached in total. These criteria were as follows:

- Related to lesson study
- Reporting a lesson study practice (not a review article, book review, etc.)
- Related to foreign/second language teacher education/ professional development
- Peer-reviewed
- One if occurring on both databases
- Not conducted in the Turkish context

After all, criteria were introduced, 11 studies were left for the focal review of international research published between 2011 and 2018. The table below summarizes the articles included in the international lesson study practice and research.

Author(s)/Year	Title	Research context
Ceallaigh, Hourigan, and Leavy (2018)	Developing potentiality: Pre-service elementary teachers as learners of language immersion teaching	Irish-medium immersion, content-based, elementary setting / pre-service teacher education
Von Esch, Kavanagh (2018)	Preparing mainstream classroom teachers of English learner students: Grounding practice-based designs for teacher learning in theories of adaptive expertise development	Linguistically diverse English language learner (ELL) context in the USA / pre-service teacher education
Cammarata and Haley (2018)	Integrated content, language, and literacy instruction in a Canadian French immersion context: A professional development journey	French immersion classes in Canada / 6th-12th grade teachers' professional development
Nashruddin and Nurrachman (2016)	The implementation of lesson study in English language learning: A case study	Teaching English as a foreign language (EFL) / High school teacher in Indonesia
Nami, Marandi, and Sotoudehnama (2016)	Computer-assisted language learning (CALL) teacher professional growth through lesson study practice: An investigation into EFL teachers' perceptions	Professional development in CALL / Iranian context
Csida and Mewald (2016)	PrimarWebQuest in foreign language education	Professional development in CALL / Austrian context
Cajkler and Wood (2016b)	Mentors and student-teachers "lesson studying" in initial teacher education	Initial teacher education of pre-service teachers / Modern languages program in the UK
Lander (2015)	Lesson study at the foreign language university level in Japan: Blended learning, raising awareness of technology in the classroom	Blended learning and technology use in foreign language education / Japanese university context
Cajkler et al. (2013)	Lesson study: Towards a collaborative approach to learning in initial teacher education?	Initial teacher education of pre-service teachers / Modern languages program in the UK
Tan-Chia, Fang, and Chew Ang (2013)	Innovating the Singapore English language curriculum through lesson study	Lesson study as a way for K-12 teachers to familiarize with recently introduced national curriculum in Singapore
Walker (2011)	How ‚language-aware' are lesson studies in an East Asian high school context?	Learning study practice on content-based language instruction in Hong Kong

Table 1: International studies included in the review

After the studies shown in *Table 1* were compiled for extensive review and analysis, the research related to lesson study practice in Turkey was searched. The last inclusion criterion was changed to "*conducted in the Turkish context*" and all criteria were introduced. In addition to Scopus and ERIC, another database *TR Dizin* was searched. TR Dizin is a database administered by *Turkish Academic Network and Information Center,* a governmental body that aims to support academic production in Turkey with information services and technologies. Arguably, TR Dizin is a way to search and find rigorous academic work published in the Turkish context but are not indexed in international databases. Search in TR Dizin resulted in 18 studies in addition to the earlier 85 studies from Scopus and ERIC. In total, three studies have been reached as illustrated in *Table 2*.

Author(s)/Year	Title	Research context
Coskun (2017)	The application of lesson study in teaching English as a foreign language	Lesson study practice at a tertiary level institute.
Yüzbaşıoğlu and Babadoğan (2016)	Recommendations of participating teachers on the application of lesson study in Turkey	K-12 context, in various school districts in an urban area.
Demirbulak (2011)	Training English language student teachers to become teacher-researchers	Initial teacher education at a university context, training teacher-researchers

Table 2: Studies conducted in Turkey and included in the review

RESULTS

International research

In this section, studies that are eligible according to our review criteria are presented in a summative way. Firstly, Ceallaigh, Hourigan, and Leavy (2018) conducted their study in an Irish-medium content-based instructional setting. They utilized lesson study to design mathematics lessons in a second language and content integration program in the elementary school context. With a qualitative case study design, they investigated the issue of how pre-service teachers can get ready by the lesson study model. The authors discussed that this "integrated mindset" supported teachers to be more language-attentive and the participants' awareness towards of the role of language use in content-based instruction increased. They suggested that lesson study provided an initial teacher education space that helped pre-service teachers balance content and language in immersion settings.

Secondly, Von Esch and Kavanagh (2018) investigated the efficiency of a modified lesson study practice that they called *EL/Math Studio*. Drawing on lesson study model, they analyzed qualitative data from teachers working in second and third-grade classrooms composed of learners with linguistically diverse backgrounds in the USA context. Their analysis demonstrated that elementary teachers working with ELLs had problems of practice in terms of teacher language, comprehensible input, lesson planning, and explaining content knowledge and concepts. In the dialogic reflections structured by their Studio model, the participant teachers engaged in the problems they experienced in a reflective and scaffolded way. The structure of this model gave opportunities for them to develop their expertise by drawing on their existing capital of professional knowledge as a group, analyzing direct data from students. The authors observed that this practice-based, situated teacher education model helped teachers integrate language and content, identify instructional problems, and design solutions for them sustainably.

Another prominent study by Cammarata and Haley (2018) also pointed out the use of lesson study in terms of integrating content, language and literacy instruction in a French immersion context in Canada. In a longitudinal single case study design, they worked with Canadian teachers working in 6th-12th grades, seeking to empower them in terms of developing their curriculum planning skills by the use of lesson study as a developmental model. They discussed that collaborative lesson and curriculum planning in lesson study model helped the participants frame their existing knowledge and associate it with their actual teaching contexts although they initially had difficulty figuring out what language and literacy components to focus on while planning content-based lessons. The authors also provided evidence of the collaborative model's potential for maximizing teacher learning. The collaborative nature of the lesson study model aligned with learning theories like situated learning and communities of practice. On the other hand, Cammarata and Haley (2018) underlined a debilitating drawback due to the gap between what was planned by teachers and what was accomplished. Even though they stated that this gap got closer at the end of the research project that lasted one and a half year, the difficulty in changing teacher beliefs was highlighted.

Within the context of *learning study*, which is a teacher professional model that is very similar to lesson study, a study by Walker (2011) in Hong Kong provided similar results. The authors included this learning study-oriented piece of research in our review, considering that both learning and lesson study models are based on the idea of teachers paying extensive focus on student learning while following a recursive and collaborative practice (Wood, 2017). Walker (2011) reported that her *learning study* practice demonstrated evidence for more "language-aware" science teachers and empowered these teachers by making them feel more competent in a content-based education context in.

Another study from the Indonesian context authored by Nashruddin and Nurrachman (2016) investigated the lesson study model in a case study design in teaching EFL at high school context. Drawing attention to the inclusive characteristic of the model, they suggested that inclusion of all ideas while co-developing lessons play an important role in the success of this model. They also proposed that focusing on student learning is of critical importance while observing the research lessons as well as reflecting on them.

Fifth, Nami, Marandi, and Sotoudehnama (2016) authored an article investigating the professional development of EFL teachers in CALL practices within the lesson study model in Iran. They analyzed the phases of the lesson study cycle implemented in the practice they investigated. As for the collaborative lesson planning phase, the results revealed that Iranian teachers had different opinions while evaluating their experience in the collaborative lesson planning phase in the model. The authors believed that this lack of consensus may have been due to the individualistic working culture in the country. Second, the participant teachers found implementing phase effective for their CALL professional development due to "direct engagement in the sustained teaching with technology based on the lesson plan they had developed" (Nami, Marandi, and Sotoudehnama, 2016: 671). Similarly, the observation phase was also valued by the participants as it enhanced their technological pedagogical knowledge. Last, the content of various types of peer-feedback was analyzed. The authors observed that the participants had occasional difficulties in providing negative feedback to their peers, which was associated with cultural aspects in the study.

Similar to the previous article, Csida and Mewald (2016) conducted a CALL professional development study in the Austrian context. In order to identify the skills required to work successfully on an instructional platform called *PrimarWebQuest*, they analysed how the participants acquired needed strategies and how their autonomy evolved in the process. Focusing primarily on the effectiveness of this platform rather than lesson study oriented professional development, the authors valued the use of lesson study as an effective way to refine teachers' lesson planning skills with regard to a specific instructional platform that is new to them.

Another CALL-related study was conducted by Lander (2015) at a university level foreign language teaching setting in the Japanese context. Based on a theoretical framework informed by

blended learning and learner autonomy, the author investigated the effect of technology on academic achievement and student attitudes. Lesson study was used as a model to aid teachers and learners to reach improved achievement through online technology. Results indicated that lesson study practice can be useful in introducing new techniques and approaches as long as it is implemented effectively. Especially the communicational framework that is fostered by the lesson study model can make transformative professional development for language teachers who are expected to keep up with evolving innovations in education.

In the next study, Cajkler and Wood (2016b) reported the relationship between mentors and pre-service teachers in a lesson study experience in modern languages in the UK. Investigation of learning outcomes of both mentors and pre-service teachers suggested that they benefited from reflective practice led by the structure of the lesson study model. The participants also discussed that their knowledge about learning and collaborative teaching increased and they developed a "pedagogic literacy", which refers to sustainable professional development through a continuum (Cajkler and Wood, 2016b: 94). Another study by Cajkler *et al.* (2013) conducted in a similar context explored the collaborative aspect of lesson study for pre-service teachers and their mentors. They used communities of practice framework to have a better understanding of lesson study experience for the stakeholders. Findings indicated that the lesson study model supported the professional development of not only pre-service teachers but also of all stakeholders including the experienced and veteran mentors and teachers.

In the Singaporean context, Tan-Chia, Fang, and Ang (2013) used lesson study as a scaffold and mediator while innovating a curriculum in English language education. They used lesson study as a tool to improve language teachers' professional capacity to carry out teaching strategies while teaching the revised version of English language teaching syllabi in Singapore. In their mixed-methods analysis, they focused extensively on the links between lesson study practice and delivery of national curriculum. The findings demonstrated that lesson study was an effective way to link teachers' subject knowledge with the new curriculum content. By connecting theory to practice in a praxis-based model, lesson study provided a structure for teachers to map and implement their theoretical knowledge. It was also reported in the study that the participant teachers' mindset shifted, in that, with the exclusive focus on learning in lesson study rather than teaching, teachers were able to abandon teacher-centric view and practices and develop an understanding of teacher who leads and facilitates learners, as projected in the new national policy.

Research in the Turkish context

Unfortunately, lesson study has not been acknowledged by language teachers, teacher educators, and administrators in Turkey, as the number of studies that reported prior practice suggests. Even though studies within the context of school subjects other than foreign/second language education are more common (e.g. Bozkurt and Yetkin-Özdemir, 2018), there were still a number of studies eligible for our review criteria.

The first study was conducted by Coskun (2017) in a context of tertiary level ELT teaching at a preparatory program. The study investigated how lesson study practice influenced EFL instruction and the professional development of three EFL instructors. Coskun asserted that observing student learning in the classroom helped EFL instructors to become more aware of problems emerging in teaching practice and show agency to solve these problems. Revising research lessons to overcome problems regarding student learning aided the participants to increase their pedagogical knowledge by learning more about classroom dynamics and learners. However, a critical concern was underlined about convincing more EFL instructors to engage in lesson study oriented professional development. Due to the existing workload at tertiary level language preparation contexts in Turkey, Coskun (2017) suggested taking precautions to maintain teacher commitment in lesson study practice. Another challenge reported by the author was Turkish EFL instructors' lack of observation experience. His case study provided diagnostic points to take into consideration in conducting effective lesson study practice in the Turkish context.

Within the Turkish context, Yüzbaşıoğlu and Babadoğan's experience (2016) highlighted lesson study as a professional development model based on constructivist teacher learning. The data were collected from socio-economically diverse school districts in a large city in Turkey. The findings showed that lesson study practice improved teacher learning, led to improved communication skills, efficiency and provided an opportunity for interaction and exchange of professional ideas. The participants were positive about lesson study as a professional development model and complained about the one-off professional development seminars provided at the beginning or end of the academic year. On the other hand, the participants asserted their concern about the sustainability of lesson study and the amount of time required in order for language teachers to integrate it into their daily teaching practice.

Demirbulak's (2011) study gave a practical instance from initial teacher education in the Turkish context. Demirbulak investigated the effects of lesson study model in training EFL teacher-researchers. In order for the pre-service teacher participants to develop their skills of planning English for Specific Purposes (ESP) lessons, the participants went through micro-teaching structured by lesson study model, and they were asked to generate a research focus and research agenda based on this experience. The findings gave evidence of increased teacher awareness and reflexivity. Engaging in reflective practice structured by lesson study supported the participants to design and revise research lessons with a critical agenda.

DISCUSSION

Drawing on the systematic review of the lesson study practices worldwide, this study aimed to highlight the potential of lesson study model to support language teacher professional development in Turkey. Our intention was to problematize the prospective affordances of lesson study for issues related to language teacher professional development in the country.

Potential of lesson study to prepare teachers for content-based language education

International review of research concerning lesson study highlights the potential of lesson study to empower content-based language education such as immersion classes in the Republic of Ireland (Ceallaigh, Hourigan and Leavy, 2018) and Canada (Cammarata and Haley, 2018) or ELLs in the USA (Von Esch and Kavanagh, 2018). The authors clearly demonstrated that lesson study's structure which allows a group of teachers to work specifically on student learning in a specific research lesson provides a recursive process to observe, revise, and develop this lesson collaboratively. This process supports teachers to counterbalance content and language so that more language-aware teachers (Walker, 2011) or communities of practice on content-based teachers (Cammarata and Haley, 2018) can emerge.

Even though it is not as common as in the European Union, content and language integrated learning (CLIL) practices do exist in Turkey, mostly in private institutions (Tok and Aribas, 2008). Also, as clearly stated in the visionary statement of educational reform in Turkey, the policy-makers are planning to adopt an interdisciplinary approach to language learning integrating content areas like math, science, social sciences, and visual sciences with language so that learners can actually use content-based language (MoNE, 2018: 68). Since the 2017-2018 academic year, the policy has been undergoing a pilot study process across Turkey in the schools administering an intensive foreign language curriculum provided by MoNE. Although international research gives a number of lesson study practices all around the world with intensive content-based and immersion language education contexts as examples of "good" and "sustainable" practices, our review shows that Turkish lesson study practice in content-based language education in K-12 contexts, which could be highly beneficial to realize the educational vision of 2023 in Turkey, has not been reported at all.

Initial language teacher education can be revisited with lesson study

Pre-service language teacher education, specifically the practicum period, was another aspect to underpin in some of the reviewed studies. Language teaching is exceptional as it requires teachers not only to know but also to perform in the language they teach so that there are particular challenges language teachers face (Mercer, 2018). In the pre-service language teacher education context in Turkey, several challenges have been reported such as lack of reflective practice and teacher reflexivity (Akcan, 2010; Tülüce and Çeçen, 2015), lack of experience to observe peers and/or to be observed by mentors/supervisors (Coskun, 2013), and lack of collaboration with peers and mentors (Merç, 2010; Eröz-Tuğa, 2013; Ceylan, Uştuk and Çomoğlu, 2017).

Considering the research reviewed, lesson study can be given as an alternative model to overcome those trajectories to a certain extent. To illustrate, Cajkler and Wood (2016b) suggested that the lesson study model promoted reflective practice both among pre-service teachers and their mentors in initial teacher education programs in the UK. Moreover, lesson study also functioned as a collaborative scaffold that supported pre-service teachers to explore the complexities of pedagogies and develop pedagogical literacy (Cajkler and Wood, 2016b: 95). Likewise, collaborative action as a result of lesson study-informed initial teacher education was reported by Cajkler *et al.* (2013). They asserted that communities of practice helped pre-service teachers to establish a link between their education, and practice and these communities of practice created pedagogical awareness both for pre-service teachers and their mentors.

Nevertheless, the authors do not propose lesson study as the "the ultimate solution" to problems of initial teacher education in Turkey because prior research also showed certain shortcomings of lesson study practice specifically in initial teacher education. In an extensive review by Larssen et al., (2018), the authors underlined two critical bottom-lines to consider in order to strengthen lesson study practice and research. First, the concept of "learning" should be very clear for all lesson study stakeholders. Therefore, forming a common ground in terms of teaching philosophies plays a critical role in the use of lesson study in initial teacher education in which pre-service teachers typically lack in pedagogical experience as a teacher. The second point they highlighted was that pre-service teachers need to better understand classroom observation practice. Within the Turkish context, observation both in terms of to observe and to be observed is a problem (Merç, 2010). Especially, mentor observation as an anxiety provoker for EFL teachers in Turkey has been reported (Aydin, 2016). Notwithstanding the problems, observing peers (Cajkler and Wood, 2016b) and mentor observation and feedback (Merç, 2010) are among key strategies in teacher education. Also, Zhang, Yuan and Liao (2018) highlighted that teachers engaging in lesson study benefit from observing research lessons greatly even though they are not the one teaching.

In the in-service context, it is also important to remember that assessment of what teachers learn from lesson study experience or their performance in it by their colleagues or administrators/mentors can provoke negative feelings. Debilitating influence of power relations can be manifested by that kind of assessment (Zhang, Yuan and Liao, 2018). To avoid this, certain precautions should be taken. Therefore, the future research should focus on the use of lesson study model while also being careful about the takeaways of the experiences reported in the prior research.

Fine-tuning in-service language teachers for policy changes and innovations with lesson study

The efficiency of the lesson study model to prepare teaching professionals for educational innovations and policy changes have been discussed in international research of lesson study practice. Studies by Nami, Marandi and Sotoudehnama (2016), Csida and Mewald (2016), and Lander (2015) demonstrated how effective lesson study-oriented professional development can assist teachers to get familiar with and adopt technological innovations and to use them in their teaching practice. As suggested by Dikilitas (2015), professional development has changed in terms of both form and content due to the swift developments of the modern world. In the context of education, more inquiry-based professional development for teachers

gained interest and momentum such as action research (Tanis and Dikilitas, 2018) and lesson study (Johnson, 2009; Tasker, 2011). In terms of introducing new technology to teachers, professional development programs in Turkey have mistakenly been detached from the actual teaching ecology (Korkmazgil and Seferoglu, 2013; Oruç-Ertürk, Gün, and Kaynardağ, 2014) making them top-down attempts to solve the problem, which is unlikely to succeed (Wyatt and Ončevska-Ager, 2016).

In tandem with Dikilitas' assertion (2015), prior studies show that lesson study is different than conventional professional development format both in content and form. For instance, Csida and Mewald (2016) problematized that the grading system of foreign language education results in a lax attitude of learners in Austria, which they argued is to be solved by promoting learner autonomy. Independent language learners actively engaging in tasks outside of the classrooms on virtual platforms like PrimarWebQuest was discussed as a solution (Csida and Mewald, 2016). However, a problem arose about how ready language teachers are to facilitate learners with this platform. In their instance, lesson study practice focused both on teachers' and learners' learning, and teachers developed themselves not only about the best ways to implement PrimarWebQuest but also to empower learners throughout lesson study.

Rather than being "trained" in educational innovation, language teachers need to situate the "content" of professional development activity that is meaningful to them. Moreover, the "form" of their action should be inquiry, practice, and process-based. Our review shows that lesson study has got the potential to fine-tune in-service teacher education in Turkey according to actual needs and to bring authenticity to teacher learning. Tan-Chia and colleagues' research (2013) in the Singaporean context supports our argument. In their study, the lesson study model helped teachers have a more skilful use of teaching strategies and formative assessment practices introduced by the current language curriculum.

MoNE's visionary report (2018: 67) asserted that language education in Turkey is planned to be supported by online and mobile technologies. Moreover, the content available at *Egitim Bilisim Agi*, an online platform that provides content and solutions for K-12 teachers, will be actively integrated into language education (MoNE, 2018: 69). In light of these visions, language teachers, teacher educators and administrators should problematize how ready they are to integrate technological innovations in their classroom practices. Prior research in the Turkish context referred to the technology as a serious stressor for teachers and a source of foreign language teaching anxiety (Aydin, 2016). Likewise, Carbová and Betáková (2013) pointed out teachers who do not participate in-service teacher education programs can feel alienated and overwhelmed by the fast development of technology professional development programs. Accordingly, lesson study practices for integrating technological innovations into teaching can be a prospective future direction for teacher education research in the country.

CONCLUSION

Our review focused on international research of lesson study practice and compared it with local practices reported in the context of foreign language teacher education in Turkey. Drawing on the educational reform envisioned by MoNE in Turkey, three major themes emerged. First, lesson study has a potential aiding EFL teacher in the K-12 context to integrate language and content in a balanced way provided that administrative support is ensured. Second, the lesson study model scaffolds initial language teacher education in that pre-service teachers can combine their education with practice in a reflective way and can grow into reflexive practitioners through lesson study. Moreover, lesson study practice can improve their collaborative working culture and pedagogical literacy as long as shortcomings of the prior practices are taken into account. Last, lesson study practice can be used to transform in-service teacher education both in terms of content and form so that teachers can engage in and develop meaningful practices. Our review discussed international research reporting a number of examples of how lesson study helped language teachers adopt technology and innovations into their teaching practice.

Our primary aim with this review paper is to provide a research agenda informed with local realities in language teacher education. Based on the ongoing discussion about the gap between researchers and practitioners within the context of applied linguistics and foreign/second language education (Ellis, 2010; Sato & Loewen, 2019), we reviewed practice-based research to synthesize takeaways in accordance with Turkey's 2023 Vision of Education and suggest that implementing lesson study practices in language teacher education can be one of the keys to success. On a broader perspective, desideratum for prospective research may include focusing on lesson study practices reported in the prior research and analyze them with a theoretical framework in the light of local practices and realities. Such "thinking allowed" papers indicating stances of researcher-practitioner collaborations in a particular local context can potentially create a praxis-based research agenda for the field of language teacher education.

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THE STRATEGY THE USE OF FALSE ASSUMPTION AND WORD PROBLEM SOLVING

Petr Eisenmann[1]
Jiří Přibyl[1]
Jarmila Novotná[2]✉

[1] Department of mathematics, Faculty of Science, Jan Evangelista Purkyně University in Ústí nad Labem, Czech Republic

[2]Faculty of Education, Charles University, Czech Republic

✉ jarmila.novotna@pedf.cuni.cz

ABSTRACT

The paper describes one problem solving strategy – the Use of false assumption. The objective of the paper is to show, in accordance with Phylogenesis and Ontogenesis Theory, that it is worthwhile to reiterate the process of development of the concept of a variable and thus provide to pupils one of the ways helping them to eliminate usual difficulties when solving word problems using linear equations, namely construction of the equations. The paper presents the outcomes of a study conducted on three lower secondary schools in the Czech Republic with 147 14–15-year-old pupils. Pupils from the experimental group were, unlike pupils from the control group, taught the strategy the Use of false assumption before being taught the topic Solving word problems. The tool for the study was a test of four problems that was sat by all the involved pupils three weeks after finishing the topic "Solving word problems" and whose results were evaluated statistically. The experiment confirmed the research hypothesis that the introduction of the strategy the Use of false assumption into 8th grade mathematics lessons (14–15-year-old pupils) helps pupils construct equations more successfully when solving word problems.

KEYWORDS

False assumption, word problems, false position method, phylogenesis and ontogenesis theory, problem solving

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Highlights

- *The study describes one of the possible ways of facilitating solving of word problems to pupils.*
- *The experiment confirmed that introducing the Use of false assumption into 8th grade mathematics lessons can help pupils construct equations more successfully.*

INTRODUCTION

Our paper focuses on the use of a once much used but nowadays overlooked solving strategy – the Use of false assumption (UFA). This strategy is rooted in ancient history (Přibyl, Eisenmann and Gunčaga, 2018). Our goal is to introduce this strategy as a suitable propaedeutic to solving word problems using linear equations. The goal of the here reported experiment conducted on three lower secondary schools in the Czech Republic was to confirm this conjecture. Pupils' ability to solve word problems efficiently is nowadays accented in national curricular documents (Jeřábek et al., 2013; National Council of Teachers of Mathematics, 2000).

Introduction to the problem

Without any doubt, mathematics education on primary and secondary school levels prepares pupils not only for solving school problems but also shows them possible ways of solving real-life situations. In everyday life, we use natural language, which is diametrically different from the artificial language of mathematics. The language of mathematics allows us to describe a problem situation unequivocally and offers us the tools for its solution. That is why it is reasonable to teach pupils to switch between the two languages at all ages. As stated in (Novotná, 2000a: 5):

> 'Most life situations are described in words. Word problems constitute one of the few school mathematics domains which require mathematization of situations described in words and the transformation of a mathematical solution back to the context of the problem.'

(Lewis and Mayer, 1987; Verschaffel, De Corte and Pauwels, 1992) and others show that solving a word problem is difficult

regardless of the solver's age. We believe this is so because word problems can be regarded as linguistic descriptions of problem situations where questions are posed and the answer is obtained by application of mathematical operations to numerical or logical data available in the problem wording.
In contrast to (Lewis and Mayer, 1987) we do not work with the inner structure of a word problem; for the needs of this study, we look at it from the outside. That is why we can classify verbally set problems into two categories: *Word problems* and *Verbally stated numerical problems* (De Corte, Greer and Verschaffel, 2000), which we do not consider to be word problems. An example of a word problem is problem number 3 and an example of a verbally stated numerical problem is problem number 1 (see section Written test).
However, in both cases, we ask for the translation of natural language into mathematical language, which is much easier in the latter case.

The brief description of the UFA strategy

The core of this strategy is based on experimenting with one primary aim – finding the solution.
The whole process of the use of the UFA strategy can be described as follows:

1. We choose a number which we think could be a possible

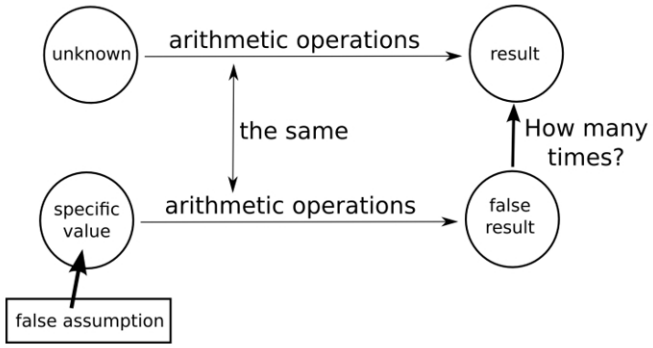

Figure 1: Schema of the process of the use of false assumption (source: authors)

Let us remark here that the UFA strategy is not universal and can only be used with a specific type of problems. It can be used successfully in solving those problems where the value of the number in the problem wording is directly proportional to the result. This means that successful use of the strategy requires from the pupil the knowledge of direct proportion and mastery of arithmetic operations with fractions. If the sought number is x and the known result y, then the relation between the two numbers is $y = kx$.
To make the process of solving a problem using the UFA strategy crystal clear, let us demonstrate this strategy on the solution of a very simple verbally stated numerical problem.
Task A *Problem:* Find the number whose result if you add its triple is 200.
Solution: Let us suppose that the desired number is for example 10 (the false assumption). Let us now carry out the operations from the problem:

$$10 + 3 \times 10$$

The result is number 40. This number is five times smaller than 200. This means the desired number must be five times greater than our original assumption. Therefore:

$$10 \times 5 = 50$$

Answer: The number is 50.
Let us now show why this procedure can be used in the solution of this problem. Let us present the schema of the whole solving process. Let us label the instruction 'add its triple to the number' as function f. This function has the character of a direct proportion. Our goal is to find such an argument whose function value equals 200. We know that $f(10) = 40$. As function f is linear and 40 is five times less than 200, then also 10 must be five times less than the unknown desired number.
Let us state here that the basic idea of the UFA strategy is relatively old. Its roots go as far as the Rhind mathematical papyrus (problems no. 24–26) (Bunt, Jones and Bedient, 1978: 30–32; Chace, Bull and Manning 1929: 141–142) and even cuneiform tablets of Babylonian mathematicians (Høyrup, 2002). For more details about history, we recommend to the reader to study the third chapter of the book (Chabert, 1999).
The same idea can be come across in many later works, for example in works of G. Cardano (Smith, 1929: 201), B. Pitiscus (Smith, 1929: 437) or Fibonacci (Boman, 2009), who referred to the strategy as *False position* or *Rule of False* (Regula Falsi). Nowadays we attach very narrow meanings to the concepts of *False Position Method* or *Regula Falsi Method*. These concern numerical solutions of equations using straight lines. However, if we inspect this strategy more attentively, we can see it has two variants. The first one is *Simple False Position* and the other *Double False Position* (Chabert, 1999: 84). While the first one corresponds to our conception, the other one corresponds to the well-known concept of numerical mathematics.

The UFA strategy in school mathematics

The UFA strategy can nowadays be come across in school mathematics only in two directions. The first accentuates the motivational aspect of the historical background of the studied issue. This can be exemplified by the work (Ofir and Arcavi, 1992), where historical resources are used to motivate pupils to think about mathematics and its potential for today. However, based on an experience Ofir and Arcavi recommend the use of the presented 'Egyptian method' only with above average pupils. The other direction focuses on the use of this method in the mathematics classroom as a tool that puts emphasis on a phylogenetic approach to the teaching of different mathematical concepts. Winicki (2000) points at the fact, that if teachers want to introduce historical elements into their lessons, they must be trained to it.

Importance of the problem

Equations are usually introduced in Czech schools in the 8th grade (14-year-old pupils). At first, pupils are introduced to linear equations, then to transformations of equations and in the end to their solution. This often goes hand in hand with the model of scales, nowadays also illustrated by computer animations (Bruder and Weiskirch, 2013). Then pupils start solving word problems using equations.
A major difficulty when solving this type of problems is to set up the appropriate equation. That is natural. Even in the history of mathematics, the introduction and use of an unknown and

thus of equation solving procedures were a slow and gradual process. For example, problems number 24–27 from the Rhind mathematical papyrus are solved using arithmetical operations with numbers. Problem number 26 gives a more detailed description of how to solve similar problems and the solver learns the solving procedure on the given example. (Chace, Bull and Manning, 1929)

The same approach can be observed in Diophantus' Arithmetica in about 250. Diophantus was able to solve relatively difficult problems. The reputed Indian mathematicians Brahmagutpa (6th century) or Bhaskara (12th century) were also able to solve very complex equations but without using the symbolic language of equations. And the same was done by al-Kwarizmi, whose work influenced European mathematics for a very long time. Works of L. Pisano, N. Chuquet or Ch. Rudolff are affected by this approach but these mathematicians were aware of the need to simplify the notation and each of them contributed to the symbolical notation of equations in their own way. The whole process culminates in the work of René Descartes, who is one of the first mathematicians introducing the concept of an unknown and symbolical recording deliberately (see e.g. Descartes, 1954). This step in development changed the approach to using equations dramatically. Equations became widespread as an appropriate instrument for problem solving.

We have proceeded from the generally accepted assumption that ontogenetic development is not independent of phylogenetic development (Brown and Heywood, 2010; Dubinsky et al., 2005). A number of authors demonstrate the relationship between the difficulties with which students are confronted during the learning process and the "breakneck" historical development within the given area of mathematics (e.g. Katz et al., 2000).

Results of Czech pupils in PISA testing when solving real-life problems are worse than the average value of other countries included in this research (Blažek and Příhodová, 2016; Palečková, Tomášek and Blažek, 2014).

Theoretical background

The starting point for our research was the Phylogenesis and Ontogenesis Theory (Furinghetti and Radford, 2002). On the grounds of this theory, we focused on the difficulties connected to the issue of solving word problems. We agree with (Stacey and MacGregor, 2000: 149) '…that a major reason for the difficulty is not understanding the basic logic of solving a problem by algebra.' As noted above, solving word problems is burdened with a wide range of difficulties. In the Czech school environment, we perceive the process of the mathematization of the problem as one of the biggest challenges. This process is characterized by Jupri and Drijvers (2016) as the fifth category of obstacles we can come across in word problem solving:

> 'Finally, the category of mathematization (MATH) concerns the difficulty to translate back and forth between the world of the problem situation and the world of mathematics, and in the process of moving within the symbolic world.' (Jupri and Drijvers, 2016: 2483)

Moreover, the whole process of word problem solving is burdened with a wide range of misconceptions (Bush and Karp, 2013).

Within the frame of Phylogenesis and Ontogenesis Theory, we focused on one of the possible ways of overcoming the above-described difficulty. The reason for proposing this approach was based on a research study in historical resources. This research dealt with strategies the *Use of false assumption* and the *Use of double false assumption* as described in detail in (Přibyl, Eisenmann and Gunčaga, 2018).

The oldest of our resources was The Rhind mathematical papyrus. We had the original source available (Chace, Bull and Manning, 1929) but we studied it in the transcription of Bunt, Jones and Bedient (1978). Let us now have a look at the assignment of the often quoted problem no. 24: 'A quantity and its seventh added together give 19. What is the quantity?' (Bunt, Jones and Bedient, 1978: 30)

The given formulation clearly suggests this is a verbally stated numerical problem. The papyrus comes from the Second Intermediate Period of Egypt, which is a long time before the algebraization of mathematics. However, the papyrus itself is not a collection of problems but something like a textbook giving also the instruction on how the problems can be solved. The following is the short transcription of the solution as presented in (Bunt, Jones and Bedient, 1978: 30–32) of the solution from the Rhind papyrus (Chace, Bull and Manning, 1929: 141–142).

Let us suppose that the sought quantity is 7. If we follow the assignment we add one seventh to the sought quantity, which is 1. Then the result is number 8, but it should be 19. How many times do I have to enlarge the original guess to get 19? Number 7 must be multiplied by the fraction $\frac{19}{8}$ and the sought number is $\frac{7\times19}{8}=\frac{165}{8}$.

If we compare this record to our definition of the UFA strategy, we come to the conclusion that this solution corresponds to the definition. The study of further resources (see the short above-mentioned historical excursion) lead us to the conclusion that the UFA strategy was around until the beginnings of the algebraization of mathematics as such. Once equations were introduced into the process of solving problems, the strategy ceased to be one of the tools of mathematicians but it gained a new role. The new role is to be looked for in textbooks of mathematics. Formerly known by the Latin name *Regula Falsi*, it got a new name (English translation) – the *False Position Method*.

The person who was greatly responsible for the inclusion of the UFA strategy into teaching materials was the important personality from the history of mathematics, Benjamin Banneker (Lumpkin, 1996). (Cajori, 1890) shows that the UFA strategy was present in mathematics textbooks until the end of the 19th century.

Another important framework for our research is the *Theory of didactical situations* (TDS) in mathematics (Artigue,

Haspekian and Corblin-Lenfant., 2014; Brousseau, 1997). This theory, among others, tells us that for each problem there is a set of prerequisite knowledge which is essential for its solution.

The UFA strategy is an example of activities that can facilitate pupils' transition from arithmetic to algebra, which has been a concern for a number of researchers (Bednarz and Dufour-Janvier, 1994; Goodson-Epsy, 1998; Linchevski and Herscovics, 1996; Novotná, 2000b). Filloy and Rojano (1989: 19) state that 'Recent researches have pointed to certain conceptual and/or symbolic changes which mark a difference between arithmetical and algebraic thought in the individual.' Based on TDS we conceived the teaching of the UFA strategy in such a way to allow pupils to discover the role of a variable on their own and to build the concept of a variable on their own. This concept is the necessary condition for the transition from arithmetic to algebra.

Research question and hypothesis

The following question was posed at the beginning of our experiment:

Research question: Is the UFA strategy a convenient propaedeutic to word problems solving using linear equations?

Based on the above stated (see section Importance of the problem) we can predict the answer to the research question by formulating our research hypothesis:

Research hypothesis: The use of the UFA strategy in the teaching of eighth graders (aged 14–15) before they are taught to solve word problems using equations will help the pupils to be more successful in constructing equations if compared to pupils who have not been introduced to the UFA strategy.

MATERIALS AND METHODS

The presented research study compares the results of pupils from experimental and control groups. Results of this study are briefly described with an emphasis on historical context in (Přibyl, Eisenmann and Gunčaga, 2018). Both experimental and control groups consisted of three classes. The inclusion of the selected class into the experimental or control groups was random. The following sections describe the preparatory stage, whose goal was to prepare the teaching experiment, to choose teachers and pupils and to conduct the teaching experiment.

Preparation of the experiment

The preparatory stage of the experiment was quite long and thoroughgoing.

In total, five seminars were held in 2013 with in-service teachers in different towns in the Czech Republic. The teachers were given problems that could be solved using the UFA strategy. Each seminar was two hours long. The teachers told us how they perceived the idea and what they predicted the results of the prepared experiment would be. In total, we discussed the experiment with about 70 lower secondary school teachers.

In the seminars, we developed a set of ten problems for the teaching experiment in collaboration with the participating teachers. All the problems were tested in five different 8th grade classes on different lower secondary schools in the Ústí region and Prague in the year 2013. The pre-test was taken by 124 pupils.

The participating teachers

Out of the 70 above mentioned teachers, 14 participated in all five seminars. Out of these 14 teachers, three were selected for the experiment. The following criteria were used for the selection:

- The teacher was planning to teach linear equations to their pupils in 2014.
- The teacher would teach mathematics in two parallel 8th grade classes simultaneously in 2014.
- The teacher had taught mathematics in these parallel classes in the previous year.
- These two parallel classes (experimental and control group) were comparable with respect to school performance in mathematics.
- The teacher would be able to cooperate with researchers regularly during the whole experiment.
- The length of the teacher's practice was at least 5 years.

All these three teachers can be described as engaged; they invest a lot of energy into their teaching and have attended in-service teacher training courses. They were paid for their work in the experiment. They were two men and one woman. The length of their teaching practice was 8, 10 and 20 years.

Research sample

The sample was formed by three pairs of eighth grade classes from three lower secondary schools in three towns in the Czech Republic. One class was randomly chosen as the experimental class, the other was the control class in each pair of classes taught by one teacher. Pupils from these classes had not been involved in the preparatory stage. In total, there were 147 pupils involved in the experiment – 72 from the three experimental classes and 75 from the three control classes. Comparability, of the experimental and control classes, was verified by a comprehensive test on topics from the previous school year, which was the same for all six classes. The results were similar in each pair of classes.

None of the classes was a specialized class. None of the classes integrated physically or mentally disabled pupils or pupils with low socioeconomic status. All the pupils were of Czech nationality with the exception of three Vietnamese and five Roma pupils. All the pupils were native speakers of Czech.

The course of the experiment

The whole experiment started in October 2014 and took 8 weeks. Its course is shown in figure 2. The teaching in the experimental class started one week before the control class. That week was used in the control class for revision of the previous topic (in two schools) or was used as a project week (in the third school).

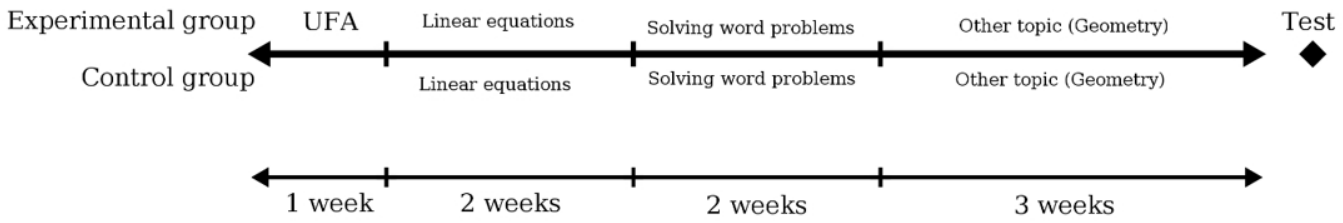

Figure 2: Timeline of the course of the experiment (source: authors)

Preparatory stage in the experimental class

At this stage, the teachers of experimental classes taught their pupils the UFA strategy. This was based on the above-mentioned set of ten problems that were presented to the pupils. The difficulty of these problems was progressive (task A forms the base of the cascade, task D is the most difficult).

When solving the first few problems for illustration, the teacher proceeded as follows: The pupils were asked to guess the result. Then they were asked to do with this guess what was required in the statement of the problem. And the teacher carried out the same with their own guess on the blackboard. Then the teacher proceeded as described in Task A. The teacher asked the pupils what should be done with the original guess to get to the number requested in the statement. This gradually led to the stage when pupils grasped the principle of solving the problem and started to be able to choose a false assumption and subsequently correct the initial guess on their own. The outcome of this stage was that at least one half of the pupils were able to solve problems such as task D. The teachers needed three lessons for this. At this stage of the experiment, the pupils were once set homework with 3 problems (before the third lesson). In the last, third lesson, a member of the research team was present in each of the three experimental classes.

Let us point out that this part of teaching can work as propaedeutic to the successful solution of problems using equations only if pupils grow aware of the fact that the choice of their guess (the false assumption) has no effect on the result of the problem, that the result will always be the same. Let us illustrate this on task A. The teachers let the pupils make the guess as they wished. Some pupils chose 5, others 1, others 20. What was important was that each group demonstrated to the others how they proceeded:

Choice 5 leads to:

$$5+3\times5=20 \quad \longrightarrow \quad 200\div20=10 \quad \longrightarrow \quad 5\times10=50$$

Choice 1 leads to:

$$1+3\times1=4 \quad \longrightarrow \quad 200\div4=50 \quad \longrightarrow \quad 1\times50=50$$

Choice 20 leads to:

$$20+3\times20=80 \quad \longrightarrow \quad 200\div80=5/2 \quad \longrightarrow \quad 20\times5/2=50$$

The fact that "the result is always the same", in other words, that the selection of the initial guess makes no difference, is the key propaedeutic element. Growing aware of this fact prepares for the concept of a variable that we introduce into the solving process of the problem that allows us to solve it using equations. This role of numbers as building blocks for construction of the concept of the unknown is discussed by Fischer (2009: 26).

The second stage in both classes

At this stage, the course of the lessons was the same in both groups and was the following. First, the pupils were introduced to solving linear equations. This took two weeks. In the remaining two weeks the pupils went on to solve word problems with the help of linear equations. At this stage of the teaching experiment, there were situations in which some pupil from the experimental group was solving the assigned word problem using the UFA method (on average 7 times in each class). In that case, the teacher checked the result and if necessary explained the mistake in the procedure and then asked the pupils to solve the problem in the way they were being taught at that time – using a linear equation. After that, the teachers moved on to a new topic (construction problems). Let us remark here the pupils had four lessons of mathematics a week in that grade.

Three weeks after teaching equations and solving word problems (see figure 2) pupils in the experimental and control groups sat the below described written test.

Written test

The test consisting of the following four problems was sat in lessons of mathematics. The teacher was present. Each pupil got one sheet of paper with statements of all four problems. There was empty space below each problem for the solution.

Each teacher got the following instruction:

- The teacher must not give any advice or explain the problem wording.
- Use of any textbooks, tables or collections of formulas is prohibited.
- Use of any calculators, computers, laptops or tablets is prohibited.
- The pupils must write the solving procedure or record the reasoning that leads to the result, not just the result.
- The pupils must write a verbal answer or double underline the result.
- The pupils have a maximum of 40 minutes for the test.

Problems:

1. One third of an unknown whole number reduced by 20% is 32. What is the unknown number?
2. The perimeter of a rectangle is 60 m. Determine the lengths of its sides if you know that they are in the ratio $7:3$.
3. Mr Hare is a successful rabbit breeder. Having bred rabbits for three years, he now has 45 rabbits. He expanded his husbandry by adding to the number of bred animals its double at the end of each year. How many rabbits did Mr Hare have at the end of his first year?

4. The price of a TV set was put up by one quarter. However, nobody wanted to buy it so the price was reduced by one half of the new price. Then it cost 10,000 CZK. What was the original price of the TV set?

All these problems can be solved using linear equations. The difficulty of the test problems is on a medium level when compared to problems used in lessons when teaching word problems solved by linear equations.

The problems for the written test were selected in such a way to make them correspond to problems the pupils had usually been solving in lessons and to involve both types of verbally given problems (word problems and verbally stated numerical problems) in the same number.

After their completion, the pupils' tests were coded. Each problem in the test was labelled by the following code:

group / number of problem / correctness of solution / way of solution / specification of the way of solution

The items of the code have the following values:

- *Group*: e – pupil is from an experimental class, k – pupil is from a control class.
- *Number of problem*: 1–4.
- *Correctness of solution*: 1 – successfully solved problem, 0 – unsuccessfully solved problem, x – pupil has not solved the problem.
- *Way of solution*: r – pupil uses a linear equation, u – pupil uses the strategy Use of false assumption, h – pupil uses another way of solution (usually heuristic strategy), n – nonsense, x – a pupil has not solved the problem.
- *Specification of the way of solution*:
  - If a pupil is unsuccessful in solving the problem (correctness of solution is 0) and uses a linear equation for its solution, then: A – if the linear equation is correctly constructed, N – if the linear equation is not correctly constructed.
  - If a pupil uses another way of solving: POK – a pupil uses the Guess–Check–Revise strategy, SE – a pupil uses the Systematic Experimentation strategy, CZ – a pupil uses the Working backwards strategy.
  - "–" (dash) in all other cases.

Examples of the coding procedure:

- Code (e / 2 / 1 / r / –) means: the pupil is from an experimental class and solved the second problem of the written test successfully, using a linear equation.
- Code (k / 1 / 0 / r / A) means: the pupil is from a control class and solved the first problem of the written test unsuccessfully. The pupil used the formulated equation correctly.
- Code (e / 3 / 1 / h / POK) means: the pupil is from an experimental class and solved the third problem of the written test successfully, using heuristic strategy Guess–Check–Revise.

Some remarks:

- The coding of all written tests was carried out by one researcher.
- No information was recorded about the pupil since we were interested in the results of the group, not individuals.
- If pupils used more ways of solution, it was coded according to the following key:
  - If a problem were solved both using linear equations and the UFA strategy or some other way of solution, then the problem was coded as solved using a linear equation.
  - If a problem were solved using the UFA strategy and some other way, then it was coded as solved using the UFA strategy.
- Having coded all problems, the results were processed by the Statistica 12 software (StatSoft, Inc., 2013).

Data analysis

To test the null hypothesis, the test of equality of two parameters of alternative distributions was used. Software Statistica 12.0 (StatSoft, Inc., 2013) was used for all statistical evaluation.

A priori analysis of test problems

In this section, the a priori analysis of the four test problems is presented. This analysis was conducted with the aim of verification of the suitability of test problems with respect to the main objective of assigning the test – to reject or fail to reject of our research hypothesis. That is why it is natural to ask about the expected possible ways of pupils' solutions of the problems.

The next two sections give a description of the results of the a priori analysis that are the same for all four problems and the results of the a priori analysis of the 3rd problem.

The a priori analysis focused on the following aspects of the problems:

- Justification of the legitimacy of the use of the UFA strategy
- Characteristics of the problem
- Knowledge and skills needed for the successful solution of the problem
- Algorithmic school solving strategy
- Solving the problem using the UFA strategy
- Overview of other possible ways that can be used for the solution of the problem
- List of possible solvers' mistakes and risks in solving the problem

Results of the a priori analysis common to all problems

With respect to the selection of problems, it may be stated that the following results of the analysis are the same for all four problems.

Justification of legitimacy of the use of the UFA strategy – each of the problems is underlain by a function expressing the dependence of the sought unknown number which has the character of direct proportion.

Characteristics of the problem – in all cases it is a two-level problem. As stated above, verbally stated problems are divided into two groups for the needs of this research study, namely: (a) verbally stated numerical problem – problems 1 and 2 and (b) word problems – problems 3 and 4.

Knowledge and skills needed for the successful solution of the problem – the following skills and knowledge are prerequisite for this kind of problems:

- mathematization of a verbally assigned text;

- arithmetical operations on natural numbers;
- arithmetical operations on rational numbers.

Overview of other possible ways that can be used for the solution of the problem – due to the experimental character of the UFA strategy, the given problems may be solved using also strategies Guess–Check–Revise and Systematic Experimentation.

List of possible solvers' mistakes and risks in solving the problem – the risks of using the UFA strategy are of general nature and are the following:

- The pupil fails to understand the relations from the problem wording and is unable to construct the expression with the false assumption.
- The pupil understands the relations from the problem wording and constructs the expression with the false assumption but does not understand the relation between the number calculated from the false assumption and the value in the problem wording.
- The pupil chooses the false assumption in a way that makes the corresponding coefficient, not a whole number, which makes further calculations much more difficult or impossible.

Results of a priori analysis of the 3rd problem

Justification of the legitimacy of the use of the UFA strategy – the function expressing the dependence of the sought unknown number from the data given in the problem assignment really has the character of a direct proportion. It is the function $f(x)$ from point 4.

Characteristics of the problem – this is a two level word problem with the same rule for calculation but a different base.

Knowledge and skills needed for the successful solution of the problem – mathematization of a verbally given text, understanding the whole and its part, arithmetical operations with natural numbers, the solution of simple linear equations

Algorithmic school solving strategy – solution using an equation:

x … the number of rabbits at the end of the first year

$x+2x=3x$ … the number of rabbits at the end of the second year

$3x+2\cdot(3x)=9x$ … the number of rabbits at the end of the third year

$f(x)=9x$ … the function describing the number of rabbits

$9x=45$

$x=5$

Mr Hare had 5 rabbits at the end of the first year.

Solving the problem using the UFA strategy – let us assume Mr Hare had one rabbit at the end of the first year. Now let us record the number of rabbits at the end of each year in table 1.

Year	New rabbits	Has
1.	–	1
2.	2	3
3.	6	9

Table 1: Solution of problem 3 using the UFA strategy (source: own calculation)

In this case, Mr Hare would have 9 rabbits at the end but he should have 45 that is five times more. This means Mr Hare had 5 rabbits at the end of the first year.

Overview of other possible ways that can be used for the solution of the problem – Guess–Check–Revise, Systematic Experimentation, Solution drawing

List of possible solvers' mistakes and risks in solving the problem –

The algorithmic way of the solution:

- The pupil does not understand the information from the problem wording and instead of adding a double of the number of rabbits they only double the number of rabbits.
- The pupil interprets the information about the increase of the number at the end of the year incorrectly and adds a double of the initial number of rabbits already in the first year.
- The pupil solves the equation incorrectly.

The use of false assumption strategy:

- The pupil does not understand the relations from the problem wording and fails to construct the expression with the false assumption.
- The pupil understands the relations from the problem wording and constructs the expression with the false assumption (the table in the model solution) but fails to understand the relation between the correctly calculated number for the false assumption and the value from the assignment. Thus they do not know how to use the calculated number.
- The pupil selects the false assumption in a way that the corresponding coefficient is not a whole number, which makes further calculations very difficult or impossible.

RESULTS

In total, 588 problems (each pupil solved 4 problems) were coded. Based on the coding we can state the following:

- All the pupils solved all the problems, that is the value "*x*" appeared in none of the codes.
- With the exception of two problems, it was always possible to identify the way of solution of the problem by the pupil. The code "*n*" appeared once in the experimental group (problem 3) and once in the control group (problem 2).
- In two problems (problems 3 and 4) (each by a different pupil), two ways of solution were used, always a solution using some other way and using a linear equation. In both cases, the equation was constructed correctly and solved successfully. According to the agreed upon key both of these occurrences were coded as "*r*". Let us remark here that problem 3 was solved in this way by a pupil from the experimental group, problem 4 by a pupil from the control group.

The following table 2 presents results with respect to the studied issue. The figure in front of the hyphen gives the number of pupils who used a linear equation when solving the problem, the figure behind the hyphen states how many of them were successful and solved the problem correctly using this way.

Problem	Group	Equation
1	Experimental	43 – 31
	Control	51 – 20
2	Experimental	40 – 28
	Control	41 – 19
3	Experimental	41 – 24
	Control	45 – 18
4	Experimental	49 – 40
	Control	55 – 27

Table 2: Solving test problems using equations ($N_E = 72$, $N_c = 75$) (source: own calculation)

The third column in table 2 shows that if pupils were solving the problem using equations, they were always more successful in the experimental groups than in the control groups. The null hypothesis about an equal probability of success in experimental and control classes was tested. The null hypothesis can be rejected with the level of significance 0.05 in all problems. The p -values for the problems are respectively: 0.0007, 0.0144, 0.0392 and 0.0002.

The following tables show frequencies of situations that occurred in the case the pupils tried to solve the given problem using equations. The number of pupils who constructed the right equation is given in columns two and three of tables 3–6. The pupils who managed to solve the equation and reach the correct result are in column two. The last column gives the number of pupils who tried to construct a corresponding equation but failed.

These tables clearly show that if pupils were able to construct a correct equation they were in most cases successful when solving it. What is crucially important here is that pupils in the experimental classes were more successful in constructing the equations. The null hypothesis on equal probability of success of pupils from the experimental and the control classes was tested again. The null hypothesis can be rejected with the level of significance 0.05 in all problems. The -values for the problems are respectively: 0.0002, 0.0127, 0.0392, and 0.0001.

For the sake of completeness, we present an overview of the overall success rate of pupils in each problem in table 7. It is quite interesting to see that some pupils from the experimental group use the UFA strategy in their solving although the strategy was introduced in lessons seven weeks before the test and they were introduced to linear equations in-between. Their numbers are given in the fifth column, where the figure in front of the hyphen gives the number of pupils who used this strategy, the figure behind the hyphen states how many of them were successful and solved the problem correctly using this way. To provide a complete view, in the sixth column we present the numbers of pupils who solved the problem in some other way, usually with a method identified as one of heuristic strategies the pupils had not been introduced to in their lessons (Working Backwards, Guess–Check–Revise or Systematic Experimentation).

Group	Constructed the right equation		Did not construct the right equation
	Resolved	Did not resolve	
Experimental	31	2	10
Control	20	1	30

Table 3: Problem 1 ($N_E = 43$, $N_C = 51$) (source: own calculation)

Group	Constructed the right equation		Did not construct the right equation
	Resolved	Did not resolve	
Experimental	28	2	10
Control	19	2	20

Table 4: Problem 2 ($N_E = 40$, $N_C = 41$) (source: own calculation)

Group	Constructed the right equation		Did not construct the right equation
	Resolved	Did not resolve	
Experimental	24	1	16
Control	18	1	26

Table 5: Problem 3 ($N_E = 41$, $N_C = 45$) (source: own calculation)

Group	Constructed the right equation		Did not construct the right equation
	Resolved	Did not resolve	
Experimental	40	3	6
Control	27	2	26

Table 6: Problem 4 ($N_E = 49$, $N_C = 55$) (source: own calculation)

Problem	Group	Total	Equation	Use of false assumption	Other way
1	Experimental	72 – 50	43 – 31	12 – 10	17 – 9
	Control	75 – 31	51 – 20	0 – 0	24 – 11
2	Experimental	72 – 44	40 – 28	16 – 10	16 – 6
	Control	75 – 27	41 – 19	0 – 0	34 – 8
3	Experimental	72 – 38	41 – 24	10 – 5	21 – 9
	Control	75 – 32	45 – 18	0 – 0	30 – 14
4	Experimental	72 – 53	49 – 40	10 – 5	13 – 8
	Control	75 – 34	55 – 27	0 – 0	20 – 7

Table 7: The ways of test problems solving ($N_E = 72$, $N_c = 75$) (source: own calculation)

DISCUSSION

The goal of the here presented study was to find out whether the introduction of the UFA strategy in mathematics lessons before teaching linear equations has a positive impact on pupils' ability to construct equations, which is imperative for solving word problems in the 8th grade.

The whole process of solving a problem has several stages. Pólya (2004) identifies these stages as (a) understand the problem; (b) devise a plan; (c) carry out the plan; (d) look back. In case of arithmetic word problems, other authors (Lewis, 1989; Morin et al., 2017) specify the stages of the solution more: (a) familiarize oneself with the problem; (b) translate the written information into a mathematical equation or system of equations; (c) solve the equation(s); (d) check one's answer in the original problem; (e) state the answer clearly in written form.

Obviously, a necessary condition for a successful solution of word problems is the construction of an appropriate equation, which corresponds to what Boonen et al. (2016: 2) state:

> 'That is, word problem solvers have to use a problem-model strategy in which they translate the problem statement into a qualitative mental representation of the problem situation hidden in the text. This mental representation subsequently allows them to make a solution plan and execute the required mathematical operations.'

Constructing an appropriate equation is not the only necessary condition. If a pupil is to construct an appropriate equation, they must be able to get the needed information from the given problem. However, research on the influence of a pupil's reading comprehension abilities has not been subject to this study.

The impact of UFA on success when constructing equations

For the needs of this research study, verbally formulated problems are divided into two basic categories, namely: (a) verbally stated numerical problems; (b) word problems. The problems included in the written test are divided as follows: problems 1 and 2 are from the category (a), problems 3 and 4 from the category (b).

Let us focus on the success rate in setting up appropriate equations. As mentioned in the methodology part of this research study, we conceive an appropriate equation as a correctly constructed linear equation whose solution bears the required result.

Table 3 shows that the success rate in constructing the equation in Problem 1 was 77% in the experimental group and 41% in the control group. Similarly, table 4 shows that pupils in the experimental group were successful when constructing the equation in problem 2 in 75% of cases while pupils in the control group only in 51% of cases. Problem 1 is a purely arithmetic problem that asks for no further knowledge, whereas problem 2 uses non-arithmetic concepts of a rectangle and its perimeter. With respect to the length of the verbal assignment in Czech, both problems were comparable. It is quite interesting to remark here (although this was not subject to the research study) that if an equation was constructed correctly, then failure in its solution was comparable in both groups (see tables 3 and 4).

Table 5 shows that the success rate in constructing the equation in problem 3 was 61% in the experimental and 42% in the control groups. Similarly, table 6 shows that the success rate of pupils in the experimental group in problem 4 was 88% while of pupils in the control group 53%. Problems 3 and 4 contained a non-mathematical (in this case pseudo-real) context. With respect to the length of the verbal assignment in Czech, both problems were comparable. Also, in this case, it holds that if an equation was constructed correctly, then failure in its solution was comparable in both groups (see tables 5 and 6).

As stated in the methodology section, the research sample and the design of the experiment were chosen in such a way to eliminate any extraneous variables as much as possible and thus to be able to focus on answering the research question fully. We can state here that the only aspect in which the groups differed was the introduction of the UFA strategy in mathematics lessons before introducing linear equations in the experimental group.

Based on the above stated results – rejection of the null hypothesis on the equal probability of success of pupils from the experimental and the control groups – we can answer the research question as follows: The use of the UFA strategy in the teaching of eighth graders (aged 14–15) before they are taught to solve word problems using equations helps pupils to be more successful in constructing equations if compared to pupils who have not been introduced to the UFA strategy.

To conclude let us point out here that this is not the only way to facilitate learning to construct appropriate equations. The research (Boonen et al., 2013; Lewis, 1989), which focuses on the analysis of problems, divides arithmetic problems into several groups. For each group, it then presents appropriate visual or mental representations that facilitate either a pupil's grasping of the problem or construction of an appropriate equation. In our research study, we put emphasis on the idea of a genetic parallel (Phylogenesis and Ontogenesis Theory) after we studied the use of the UFA strategy in the solution of problems in the past, and reiterated this process with our pupils.

The UFA strategy and other ways of transition from arithmetic to algebra

Most publications on the UFA strategy focus on its use in teacher education, especially in the context of showing historical solving procedures to teachers. For example Winicki (2000) describes a workshop for teachers in which the UFA strategy was used as a tool for getting to understand the solution of linear equations. The target group were teachers who did not teach their pupils to solve linear equations and functions; in their case, it was essential that they should understand mathematical principles because they were teaching e.g. proportion and proportionality. The participants of the workshop focused on the questions such as for what type of equations the UFA strategy was a suitable strategy, whether the result depended on the selected assumption and what advantages the participants could see in its use.

In our research, we followed a different path. The UFA strategy was taken as a tool facilitating pupils' transition from arithmetic to algebra in word problem solving.

Let us remark here that the UFA is not the only tool supporting understanding of solution of problems on linear equations. Other heuristic strategies, studied for example in (Eisenmann et al., 2015) suitable for these ends are the strategies Guess–Check–Revise, Systematic Experimentation and Solution drawing (in our case a line segment legend (Novotná, 2000b)). Table 8 gives an overview of some advantages and disadvantages of the given strategies in the facilitation of solution of word problems based on a linear equation.

	UFA	Guess–Check–Revise, Systematic Experimentation	Use of line segments
Applicability	Only for problems of the type direct proportion For problems of discrete and continuous character	For a wide range of problems of discrete character In case of problems of continuous nature no solution may be found	For a wide range of problems of discrete character, suitable especially for problems on division of the whole into unequal parts
Speed of finding a solution	A very fast procedure: Having calculated the result for the false assumption, a coefficient may be determined by which the original false assumption is multiplied.	It can be very protracted and numerically demanding, it depends on how far the first guess was from the final result.	Illustrative procedure that brings the solver to the result relatively quickly if the solution drawing is sketched well. Note: Although there might be no letters in the solution, the procedure is of algebraic nature, the unknown is represented by a line segment.
Grasping the role of an independent variable	Yes	Yes	Yes
Spontaneity of choice of strategy	Requires introduction of the strategy, its spontaneous use without prior experience cannot be expected	Often chosen spontaneously	Requires introduction of the line segment model, spontaneous use without prior experience cannot be expected
Possible problems	Use in situations that are not the type of direct proportion If the assumption is not opportune, the coefficient may be a non-whole number	Very long calculations resulting in a loss of motivation to solve the problem until the end In case of successful discovery of the solution, pupils may start using the strategy as an algorithm and lose motivation to use equations	Wrong depiction of relations In case of successful discovery of the solution, pupils may start using the strategy as an algorithm and lose motivation to use equations

Table 8: Comparison of the UFA strategy with Guess–Check–Revise, Systematic Experimentation and Solution drawing (source: own research)

We are fully aware of the fact that heuristic strategies are not the only way of facilitating the transition from arithmetic to algebra. The background of the whole process is based on a generalization of arithmetic thinking up to the level of algebraic thinking and reasoning. Some heuristic strategies may be introduced already at primary school level (Jacobs et al., 2007), which then makes the transition from arithmetic to algebra easier at the secondary school level.

A posteriori analysis of test problems

Justification of the legitimacy of the use of the UFA strategy and characteristics of the problem – as shown in the a priori analysis, all problems in the test were chosen appropriately. They are two level problems, all of which have a linear dependence in the background. The problems were also divided into two groups according to their wording – verbally stated numerical problems 1 and 2, and word problems 3 and 4.
Knowledge and skills needed for the successful solution of the problem – The pupils had all the knowledge and skills needed for the successful solution of the problems. The pupils had already been introduced to rational numbers. Mathematization of verbally stated problems was the topic of the lessons within the frame of the teaching experiment in progress.
Algorithmic school solving strategy – the anticipated algorithmic school solution of the given problems corresponded to the use of linear equations that the pupils had to construct. The practice of solving linear equations was the topic of the lessons within the frame of the experiment in progress. Let us remark here that if the pupils constructed the appropriate equation correctly, they were almost always successful in its solution. In each of the problems, there were one to three pupils who had failed to solve the problem despite having constructed the equation correctly. This was independent of the group they belonged to (see tables 3 to 6).
Solving the problem using the UFA strategy – although this was not subject to the here presented research, pupils from the experimental group had the chance to solve the given problem using the UFA strategy. Table 7 shows that this actually happened in each of the test problems. It is important to note that the use of the UFA strategy did not necessarily bring the successful solution to the problem. Analysis of pupils' solutions shows that pupils who used the UFA strategy for the solution of the given problem did not even try to construct the appropriate equation, in other words, the strategy became for them an equivalent to the algorithmic way of the solution.
Overview of other possible ways that can be used for the

solution of the problem – table 7 shows that pupils solved the given problems also using other ways than using linear equations or the UFA strategy. From our research (Eisenmann et al., 2017) we know there are heuristic strategies whose occurrence is spontaneous and that can be come across without having been taught. In some cases, their use is encouraged by the nature of the problem itself.

In our a priori analysis, we predicted the occurrence of experimental strategies Guess–Check–Revise and Systematic Experimentation. While the first strategy was to be met in the solutions of all four problems, the other was not used for the solution of any of the problems. A strategy used unexpectedly was Working backwards, namely in the solutions to problems 1, 3 and 4.

In the strategy Guess–Check–Revise we first check whether we have a solution and if not we make a revision which we believe will take us closer to the goal – in other words, we conduct a new experiment. And we carry on doing this until we get the solution. This strategy is universal and is used frequently in many different types of problems from various areas of mathematics.

The idea of Systematic experimentation is based on conducting experiments in an organized way in which each following experiment takes us "closer" to the desired goal. The strength of this strategy is even more apparent if we use computer technology. First, we make a key to how to conduct the experiments and then using a computer (most often spreadsheet) we conduct a whole series of experiments. The strategy is limited by the creation of the right key.

The strategy Guess–Check–Revise and Systematic experimentation in solving problems from school mathematics are discussed in detail in (Břehovský et al., 2013; Novotná, Eisenmann, and Přibyl, 2014; Novotná et al., 2014).

Working Backwards is a strategy commonly used in mathematics in situations when we know the "result" of the problem and are looking for the solving procedure (for example we know the initial state) or for the initial state (we know the solving procedure). An example of the first are problems from geometric construction, an example of the latter are problems in which we look for inverse operations to the given operations. Pupils involved in the experiment (both control and experimental groups) were not trained to be problem solvers for any considerable period of time and thus can be perceived as novices in the area. As Gick (1986) states it is exactly this group of solvers who tend to work backwards if the nature of the problem allows it.

Solvers' mistakes in solving the problem – apart from the mistakes mentioned in case of solution using the UFA strategy and algorithmic way of solution, the biggest problem is still the process of the mathematization of the problem as such, that is the construction of an appropriate equation, see tables 3 to 6. An expected phenomenon was that pupils from the control group would be less successful in the construction of equations, which actually happened. However, the number of pupils from the experimental group who failed to construct the equations correctly was far from negligible. Let us look at each of the problems in more detail.

In case of problems 1 and 2 (verbally stated numerical problem), the number of pupils in the experimental group who failed to construct the correct equations is one quarter of those who tried to construct it. The analysis of the pupils' solutions shows that in case of problem 1 this is caused by the high number of consecutive arithmetic operations that are related to the whole. In case of problem 2, the difficulty comes out of the fact that pupils labelled the two sides of the rectangle by two different variables, which made them construct an equation with two variables or a system of equations, which they were not able to solve.

In case of problems 3 and 4 (word problems), the situation is different. In case of problem 3, the number of pupils in the experimental group who failed to construct the correct equations equals 39% of those who tried to construct it. That is why we present a full a priori analysis of this problem. In case of problem 4, the situation is the opposite and the number of pupils in the experimental group who failed to construct the correct equations equals 12% of those who tried to construct it. We believe this high success rate can be ascribed to the small number of arithmetic operations. The mistakes that could be come across in the construction of equations were in most cases caused by wrong use of the basis.

Pitfalls in the use of the UFA strategy

As a posteriori analysis showed, a number of pupils used the UFA method while solving the problems in the written test. That is why we should ask whether the UFA strategy is an appropriate tool for the solution of arithmetic word problems. We are fully aware of the fact that the introduction of this strategy in mathematics lessons bears some risks.

Problems of non-linear nature

The first pitfall in the use of the UFA strategy is that a pupil must be able to tell whether the particular problem may or may not be solved using this strategy. In other words, they must be able to tell whether there is a direct proportion in the background of the problem. The following two problems illustrate inappropriate uses of the UFA strategy in two different cases.

Task B

Problem: The interest rate on investment into the business of mafia is 100 percent a year. I deposit 1 million. In how many years will I have 16 million on my bank account?

Solution (wrong): Let us suppose that there will be 16 million on the bank account in 3 years (the false assumption). This means I have 2 million after 1 year, 4 million after two years, and 8 million after 3 years. This is not enough; I need 16, which is twice as much. This means I have to double my initial guess.

Answer: I will have the required sum in 6 years.

Clarification: If this result is tried and tested it proves to be incorrect. The problem is not based on a direct proportion but on an exponential function.

Task C

Problem: 591 pupils are enrolled in three clubs that take place at the same time. The basketball club is attended by three times more pupils than the gymnastics club. 114 more pupils go to the swimming club than to the basketball club. How many pupils are enrolled in the gymnastics club?

Solution (wrong): Let us suppose that it is 40 pupils that go to the gymnastics club (the false assumption). Then 120 go to the basketball club and 234 to the swimming club. This makes it 394 pupils altogether, which is not enough. Thus I must multiply the initial guess by $\frac{591}{394}$.

Answer: 60 pupils go to the gymnastics club.
Clarification: If we look at the solution quickly, it may not seem as an inappropriately used strategy. However, if the result is tried and tested, it proves to be incorrect. The problem is not based on a direct proportion but on a linear function $= kx + q$, where q in a nonzero number.
Problems of other than linear nature were not set to the pupils in our research study. For the needs of our experiment, we take the UFA method only as a convenient propaedeutic to word problem solving using linear equations, not as an alternative way of solving word problems.

The risks of selection of the false assumption

Another pitfall in the use of the UFA strategy is the choice of the false assumption. An unfortunate selection of the guess can become an obstacle for the solver in case they have to multiply their guess by a non-whole number. This phenomenon can be illustrated on the solution of the following problem.
Task D
Problem: The price of a book was reduced at first by 40%, and one month later to one half of the new price. The book now costs 150 CZK. What was its original price?
Solution: Let us suppose the original price was 1 000 CZK. If we reduce the price as described above, we get the following:

$$1000 - 400 = 600$$
$$600 \div 2 = 300$$

The price after the reduction is 300 CZK in this case. To get the final price 150 CZK we have to divide our original guess by two.
Answer: The book originally costs 500 CZK.
If the guess were for example 400 CZK, the calculation would be as follows:

$$400 - 160 = 240$$
$$240 \div 2 = 120$$

Number 120 is too little, the result should be 150. So we must increase the original guess. How much? It must be directly proportional, that is:

$$400 \times \frac{150}{120} = 500$$

The choice of the guess can have a major influence on success in the solution of the problem. Already the lessons conducted within the preparatory stage of the experiment showed that some pupils tended to fail if they had to multiply the guess by a non-whole number. The participating teachers stated unanimously that this was an obstacle to mastering the UFA method for about one half of the pupils. However, it did not take long before pupils got the feel for a good choice of the initial guess, which is illustrated in the authentic solution of problem 1 from the written test in figure 3.

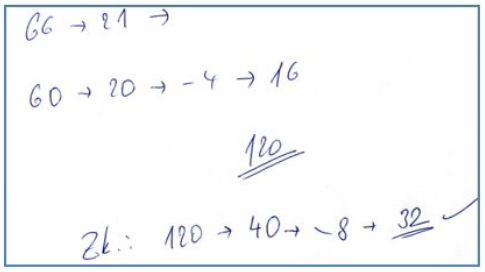

Figure 3: A pupil's solution illustrating a good choice of the initial guess. Problem: One third of an unknown whole number reduced by 20% is 32. What is the unknown number? (source: pupil's solution) Note: Zk stands for Verification.

The solver, having conducted the first operation from the assignment, dropped their initial guess (66) and replaced it by a more opportune one (60).
Let us go back to one of the above-mentioned pitfalls of the use of the method UFA. The anticipated obstacle of inopportune choice of the false assumption that requires multiplication by a non-whole number and results in a pupil's failure to finish the solution could really be come across in the tests. However, the occurrence of the phenomenon was not very frequent: 1, 0, 3, 2 (starting by problem 1). This phenomenon will not be further interpreted, as the main reason for including the UFA strategy into teaching was not to give the pupils an extra way of solving word problems but to enable them to construct equations with more ease in their future solution of word problems.

Limitations of our research

To conclude this section let us have a look at some limitations we are aware of. These limitations have to be taken into account when interpreting the results or when planning a new research study.

Research sample

We selected three pairs of classes according to the chosen teacher using the above listed criteria. Let us now discuss the limitations this selection bears:
With respect to the selected criteria, the teachers are regarded as comparable. However, there might be differences between them – both in the degree of their involvement and quality of teaching. We regard these differences as the first limitation of our study. "Comparability" of the involved teachers is based on the fact that we got to know them in the course of the one year work of posing and testing problems for the teaching experiment at the preparatory stage.
Another limitation is the "comparability" of the six classes. This "comparability" was assessed based on their school performance, which was stated according to school performance in mathematics at the end of the 7th grade. Moreover, comparability of the experimental and control classes was assessed by a comprehensive test of knowledge and skills from the previous year, which was the same in all the six classes. The results were very similar in each pair of the classes.
With respect to the generalization of our hypothesis, we are

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limited by the size of our research sample of 147 pupils (72 pupils in the experimental and 75 pupils in the control groups). However, for statistical evaluation of our hypothesis, the size of the sample is sufficient with the 5% level of significance.

Written test

Selection of only one tool for evaluation of the research hypothesis may also be seen as a limitation. This tool was the written test. However, we are convinced that for the needs of verification of our research hypothesis this tool is sufficient.

Used strategies

All the problems in the written test can be solved in other ways than using linear equations. The Working Backwards strategy can be used successfully in problems 1, 3 and 4, the Guess–Check–Revise strategy in all the used problems. This might be a possible obstacle in the evaluation of the research hypothesis. However, if exclusively word problems that can only be solved using equations and cannot be solved in any other way had been used in the test, it would have been distortion of reality in the sense that word problems suitable for practicing the use of equations very often encourage spontaneous use of the Working Backwards and Guess–Check–Revise strategies (Eisenmann et al., 2017).

CONCLUSIONS

The here presented study describes one of the possible ways of facilitating solving of word problems to pupils. The study conducted with the research sample confirms the research hypothesis that the introduction of the UFA strategy before actually teaching the process of solving word problems can help pupils construct equations.

In accordance with the Phylogenesis and Ontogenesis Theory, we tried to show that it is worthwhile to reiterate the phylogenetic development of the concept of a variable with pupils in lessons.

It is generally agreed on that the greatest difficulty in the solution of word problems at schools is their mathematization. The process of the mathematization of a word (numerical) problem is based on the recognition of three types of information present in the problem wording. These are – usually numerical values or descriptions of variables, relations between data from the assignment, and questions. It is quite easy to discern the question in the problem wording. What is really difficult is to pinpoint the variables. Only after stating the variables and unknowns is it possible to study the relations among them.

The formation of the concept of a variable is important for the transition from arithmetic to algebra. It is crucially important that pupils should grow aware of the fact that the choice of the guess (the false assumption) has no impact. If pupils realize this they are more likely to grasp the concept of a variable that must be introduced to the solution of a problem if we want to construct an equation.

Further research and open questions

Further research should focus on several different areas. The first one is the relation of the reading comprehension ability and the UFA strategy and their impact on the success of constructing equations. The research question could be whether pupils with a lower level of reading comprehension skills will be equally successful when constructing equations having been introduced to the UFA strategy as pupils with a higher level of reading comprehension skills.

We did not work with pupils as individuals in this study. They were a part of the whole and the groups were perceived as relatively homogenous. Another area in which the research could be developed is the area of individual pupils with respect to their school performance in mathematics and mother tongue and their ability to construct equations, both in the experimental group (that is having been taught the UFA strategy) and in the control group.

The results of further research should be supported by more methods than only analysis of pupils' written tests. This analysis could be supplemented by structured interviews with teachers and a questionnaire survey with pupils.

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